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SNELL LAW FOR AN ELECTROMAGNETIC PULSE TRAVELLING OVER A TORUS SURFACE AND SCATTERED BY A FLAT

LEI DE SNELL PARA UM PULSO ELETROMAGNÉTICO QUE SE PROPAGA SOBRE A SUPERFÍCIE TOROIDAL E É ESPALHADO POR UMA INTERFACE PLANA

LEY DE SNELL PARA UN PULSO ELECTROMAGNÉTICO QUE SE PROPAGA SOBRE LA SUPERFICIE TOROIDAL Y ES DISPERSADO POR UNA INTERFAZ PLANA

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J. M. Velázquez-Arcos[1], J. Granados-Samaniego[2], A. Cid-Reborido[3], A. Pérez-Ricardez[4], C. A. Vargas[5], S. Alcántara-Montes[6]

ABSTRACT

We apply a new tool that is the Maxwell-Fredholm equations, to study the collision of an electromagnetic pulse with a flat surface of separation between the two parts of a torus-shaped artefact with a highly anisotropic refractive index that forces it to move over its surface. We find that it is possible to establish a relationship between the different angles involved depending on the parameters of the toroidal coordinates, which we call a particularization of Snell's law.

Keywords: Maxwell-Fredholm Equations. Left-Handed Media. Toroidal Symmetry. Electromagnetic Resonances.

RESUMO

Aplicamos uma nova ferramenta, as equações de Maxwell-Fredholm, para estudar a colisão de um pulso eletromagnético com uma superfície plana de separação entre as duas partes de um artefato em forma de toro, com um índice de refração altamente anisotrópico que o força a se mover sobre sua superfície. Constatamos que é possível estabelecer uma relação entre os diferentes ângulos envolvidos, dependendo dos parâmetros das coordenadas toroidais, o que chamamos de uma particularização da lei de Snell.

Palavras-chave: Equações de Maxwell-Fredholm. Meios Canhotos. Simetria Toroidal. Ressonâncias Eletromagnéticas.

[1] Basic Sciences Department. Metropolitan Autonomous University. Mexico. E-mail: jmva@azc.uam.mx
[2] Basic Sciences Department. Metropolitan Autonomous University. Mexico. E-mail: jgs@azc.uam.mx
[3] Basic Sciences Department. Metropolitan Autonomous University. Mexico. E-mail: acr@azc.uam.mx
[4] Basic Sciences Department. Metropolitan Autonomous University. Mexico.
E-mail: alejandroperezr@yahoo.com
[5] Basic Sciences Department. Metropolitan Autonomous University. Mexico. E-mail: carlovaxqgmail.com
[6] Basic Sciences Department. Metropolitan Autonomous University. Mexico.
E-mail: sammotion@yahou.com

RESUMEN
Aplicamos una nueva herramienta, las ecuaciones de Maxwell-Fredholm, para estudiar la colisión de un pulso electromagnético con una superficie plana de separación entre las dos partes de un artefacto con forma de toro, con un índice de refracción altamente anisotrópico que lo obliga a moverse sobre su superficie. Encontramos que es posible establecer una relación entre los diferentes ángulos involucrados según los parámetros de las coordenadas toroidales, lo que llamamos una particularización de la ley de Snell.

Palabras clave: Ecuaciones de Maxwell-Fredholm. Medios Zurdos. Simetría Toroidal. Resonancias Electromagnéticas.

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1 INTRODUCTION

On this work we return to the use of the Maxwell-Fredholm equations to describe a highly anisotropic media which is capable to force an electromagnetic pulse to travel over the surface of a toroid surface. But now, we suppose that suddenly, the soft continuous change in the refraction index becomes completely different from a plane that cuts the torus transversally. We then have that one half of the torus is different to the other part. As the name of the paper suggests, we analyse the effect of the change in the pulse trajectory due to their passage from one section to the other of the torus which leads to a geometry-dependent Snell law. In order to make clear the tools we are using, we do a review of the toroidal coordinates and a justification for the use of the Maxwell-Fredholm equations because the latter are based on the homogeneous Fredholm equations. Then we divide the problem of the scattering in two parts, first we suppose that the pulse approaches the interface between the two different donut sectors and then we suppose that the scattered pulse leaves the interface. It is obvious that the

problem treated with conventional tools is an inhomogeneous problem but is for this reason that we expose the feasibility of using our approach. Toroidal coordinates have a wide application, particularly in the search for new energy sources and tokamaks [see references (2) and (3)], but also in flux coordinates, so the choice of a toroidal geometry is in itself interesting. The Maxwell-Fredholm equations are written as

$$rot\mathbf{E}_e(\omega) = -i\omega\boldsymbol{\mu}e^{-ih(\omega_e)}\mathbf{K}^{(\smile)}(\omega)\mathbf{H}_e(\omega) \qquad (1)$$

$$rot\mathbf{H}_e(\omega) = i\omega\boldsymbol{\varepsilon}e^{-ih(\omega_e)}\mathbf{K}^{(\smile)}(\omega)\mathbf{E}_e(\omega) \qquad (2)$$

$$\eta_e(\omega) = e^{ih(\omega_e)} \qquad (3)$$

2 THE UNITARY VECTORS $\hat{\mathbf{e}}_\sigma$, $\hat{\mathbf{e}}_\tau$ AND $\hat{\mathbf{e}}_\varphi$

In this section we will find the expressions of the unit vectors in toroidal coordinates since they will be fundamental for our work since thanks to them we will be able to easily impose the conditions that the electromagnetic fields must satisfy. So, the unitary mutually perpendicular basis that are represented in Fig. 1 is:

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$$\hat{e}_\sigma = \frac{1}{h_\sigma}\frac{\partial \mathbf{r}}{\partial \sigma} = \frac{(\cosh\tau - \cos\sigma)}{a}\frac{\partial \mathbf{r}}{\partial \sigma}$$

$$= \frac{1}{\cosh\tau - \cos\sigma}\Big[-\sin\sigma\sinh\tau\left(\cos\varphi\hat{e}_x + \sin\varphi\hat{e}_y\right)$$

$$+\left(\cos\sigma\cosh\tau - 1\right)\hat{e}_z\Big] \quad (4)$$

$$\hat{e}_\tau = \frac{1}{h_\tau}\frac{\partial \mathbf{r}}{\partial \tau} = \frac{(\cosh\tau - \cos\sigma)}{a}\frac{\partial \mathbf{r}}{\partial \tau}$$

$$= \frac{1}{\cosh\tau - \cos\sigma}\Big[\left(1 - \cosh\tau\cos\sigma\right)\left(\cos\varphi\hat{e}_x + \sin\varphi\hat{e}_y\right)$$

$$-\sinh\tau\sin\sigma\hat{e}_z\Big] \quad (5)$$

$$\hat{e}_\varphi = \frac{1}{h_\varphi}\frac{\partial \mathbf{r}}{\partial \varphi} = \frac{\cosh\tau - \cos\sigma}{a\sinh\tau}\frac{\partial \mathbf{r}}{\partial \varphi}$$

$$= -\sin\varphi\hat{e}_x + \cos\varphi\hat{e}_y \quad (6)$$

3 BEAM DEFLECTION IN TOROIDAL SYMMETRY

In order to pose the shock of the electromagnetic pulse traveling on the surface of a toroid against a flat surface, we will use a new approach, the use of the Maxwell-Fredholm equations.

Let us take equation (2) and make in the left-hand term

$$rot\mathbf{H}_e'(\omega) = i\omega\boldsymbol{\varepsilon}'\mathbf{E}_e'(\omega) \quad (7)$$

So we arrive to the equation

$$i\omega\mathbf{E}_e'(\omega) = i\omega\boldsymbol{\varepsilon}e^{ih(\omega_e)}\mathbf{K}^{(\circ)}(\omega)\mathbf{E}_e(\omega) \quad (8)$$

Now, we suppose that the electric field points toward the unitary vector $\hat{e}_\tau$ that implies the equation (8) becomes

$$\mathbf{E}_e'(\omega) = \boldsymbol{\varepsilon}e^{ih(\omega_e)}\mathbf{K}^{(\circ)}(\omega)\,\hat{e}_\tau E_\tau(\omega) \quad (9)$$

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Defining the permittivity tensor

ε (10)

In principle, there is a dependence on the frequency but, for convenience, we bequeath this to the kernel, in order to easy look the contribution of the tensor ε, which operates on the column vectors in the (σ,τ,φ) space bending the beam trajectory, then by using a similar description as the used for the Euler angles:

$$\boldsymbol{\varepsilon} = \varepsilon \begin{bmatrix} \cos 2\varphi \cos\beta & \cos\beta \sin 2\varphi & -\sin\beta \\ -\sin 2\varphi & \cos 2\varphi & 0 \\ \sin\beta \cos 2\varphi & \sin\beta \sin 2\varphi & \cos\beta \end{bmatrix} \quad (11)$$

The total effect is to make that from an initial point $(\sigma_{\circ},\tau,\varphi_{\circ})$, the parameter tour to the new value $\cos\beta\cos 2\varphi\sigma_{\circ} + \cos\beta\sin 2\varphi\tau - \sin\beta\varphi_{\circ}$, tour to the new value $\sin\beta\cos\varphi\sigma_{\circ} + \sin\beta\sin 2\varphi\tau + \cos\beta\varphi$ and τ preserves his value that is the same toroidal surface as long as:

$$\cos\varphi = \frac{1 \pm \sqrt{\frac{2\sigma_{\circ}}{\tau} + \left(\frac{\sigma_{\circ}}{\tau}\right)^2}}{1 - \frac{\sigma_{\circ}}{\tau}} \quad (12)$$

We have used the notation for distinguish between the two successive rotations over the (σ,τ,φ) space.

In terms of this last tensor, equation (9) can be written

$$\mathbf{E}'_e(\omega) = e^{ih(\omega_e)} \boldsymbol{\varepsilon} \mathbf{K}^{(\circ)}(\omega) \hat{e}_\tau E_\tau(\omega) \quad (13)$$

For simplicity, we propose that we have only two punctual emitters with the kernel given by

$$\mathbf{K}^{(\circ)} = \begin{bmatrix} {}^{1}\mathbf{K}^{(\circ)} & \mathbf{0} \\ \mathbf{0} & {}^{2}\mathbf{K}^{(\circ)} \end{bmatrix} (14)$$

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On matrix (14) the elements are

$$ {}^{1,2}\mathbf{K}^{(\circ)} = \begin{bmatrix} \frac{\sin\left[(\omega-\omega_p)\delta\right]}{(\omega-\omega_p)\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (15) $$

As we have said, we suppose that the electric field at the two initial points only have a component and indeed are identical, for example

$$ E_\tau(\mathbf{r}_1) = E_\tau(\mathbf{r}_2) = E_0 \cos(\omega_0) \quad (16) $$

In equation (7) we impose the condition that the $rot\mathbf{H}_e{}'(\omega)$ does not have σ or φ components. Also, for the magnetic field we suppose that

$$ H_\tau{}' = 0 \quad (17) $$

And

$$ H_\varphi{}' = 0 \quad (18) $$

Which means that $H_\sigma{}'$ satisfy the partial differential equation:

$$ \frac{\partial\left(H_\sigma{}' \frac{a}{\cosh\tau-\cos\sigma}\right)}{\partial\tau} = 0 \quad (19) $$

$$ \frac{a}{\cosh\tau-\cos\sigma}\frac{\partial H_\sigma{}'}{\partial\tau} + H_\sigma{}' \frac{\partial\left(\frac{a}{\cosh\tau-\cos\sigma}\right)}{\partial\tau} = 0 \quad (20) $$

But then we can write that

$$ \frac{\partial H_\sigma{}'}{\partial\tau} = -\frac{\sinh\tau}{\cosh\tau-\cos\sigma} H_\sigma{}' \quad (21) $$

And from this equation we have that the magnetic field has the form

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$$H'_{\sigma} = C_0(\tau,\varphi)\frac{1}{\cosh\tau - \cos\sigma} \qquad (22)$$

Now from equations (13-16) we can write:

$$i\omega\mathbf{E}'_{e}(\omega) = i\omega\varepsilon e^{ih(\omega_e)}\mathbf{K}^{(::)}(\omega)\mathbf{E}_{e}(\omega) \qquad (23)$$

And then it follows that:

$$\mathbf{E}'_{e}(\omega) = \boldsymbol{\varepsilon} e^{ih(\omega_e)}\mathbf{K}^{(::)}(\omega)\hat{e}_{\tau}E_{\tau}(\omega) \qquad (24)$$

Or

$$\mathbf{E}'_{e}(\omega) = e^{ih(\omega_e)}\boldsymbol{\varepsilon}\mathbf{K}^{(::)}(\omega)\hat{e}_{\tau}E_{\tau}(\omega) \qquad (25)$$

Remembering that $\mathbf{r}_1$ we are supposing that is very near to $\mathbf{r}_2$, so we can write that (although it is not strictly necessary):

$$E_{\tau}(\mathbf{r}_1) = E_{\tau}(\mathbf{r}_2) = E_0\cos(\omega_0) \qquad (26)$$

And writing explicitly the kernel:

$$\mathbf{E}'_{e}(\omega) = \boldsymbol{\varepsilon} e^{ih(\omega_e)}\begin{bmatrix} {}^{1}\mathbf{K}^{(\circ)} & 0 \\ 0 & {}^{2}\mathbf{K}^{(\circ)} \end{bmatrix}\hat{e}_{\tau}E_{\tau}(\omega) \qquad (27)$$

We can then write for the electric field by using equations (22-27) together with the explicit form of the curl in toroidal coordinates that is:

$$\nabla\times\mathbf{F} = \hat{e}_{\sigma}\frac{(\cosh\tau-\cos\sigma)^2}{a^2\sinh\tau}\left(\frac{\partial\left(F_{\varphi}\frac{a\sinh\tau}{\cosh\tau-\cos\sigma}\right)}{\partial\tau} - \frac{\partial\left(F_{\tau}\frac{a}{\cosh\tau-\cos\sigma}\right)}{\partial\varphi}\right)$$

$$+\hat{e}_{\tau}\frac{(\cosh\tau-\cos\sigma)^2}{a^2\sinh\tau}\left(\frac{\partial\left(F_{\sigma}\frac{a}{\cosh\tau-\cos\sigma}\right)}{\partial\varphi} - \frac{\partial\left(F_{\varphi}\frac{a\sinh\tau}{\cosh\tau-\cos\sigma}\right)}{\partial\sigma}\right)$$

$$+\hat{e}_{\varphi}\frac{(\cosh\tau-\cos\sigma)^2}{a^2}\left(\frac{\partial\left(F_{\tau}\frac{a}{\cosh\tau-\cos\sigma}\right)}{\partial\sigma} - \frac{\partial\left(F_{\sigma}\frac{a}{\cosh\tau-\cos\sigma}\right)}{\partial\tau}\right) \qquad (28)$$

From these expressions we can write for the electric field

$$E'_{\tau}(\mathbf{r}_{1,2}) = \frac{1}{\varepsilon'}(\frac{\partial C_0(\tau,\varphi)}{\partial\varphi})\frac{1}{\sinh\tau} = \varepsilon e^{ih(\omega_e)}\frac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta}\cos(\omega_0)E_0 \quad (29)$$

And because is a function of then:

$$E_0(\sigma_{1,2},\tau,\varphi_{1,2}) = D_0(\tau,\varphi)\frac{1}{\sinh\tau} \quad (30)$$

And substituting on equation (29) we have also:

$$\frac{1}{\varepsilon'}(\frac{\partial C_0}{\partial\varphi})\frac{1}{\sinh\tau} = \varepsilon e^{ih(\omega_e)}\frac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta}\cos(\omega_0)D_0\frac{1}{\sinh\tau} \quad (31)$$

Finally we obtain the condition:

$$\frac{1}{\varepsilon'}(\frac{\partial C_0}{\partial\varphi})\frac{1}{\sinh\tau} = \varepsilon e^{ih(\omega_e)}\frac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta}\cos(\omega_0)D_0E_0 \quad (32)$$

From equation (22) we can see that for every set (σ,τ,φ) we have a different value of C_0 and we need since equations (26-32) to establish the value of the electric field. We must remember that even the beam is moving over a torus of fixed τ, intersects a continuum of different spheres. The process is illustrated in Figure 1 where the curl trajectory is represented in yellow and the basis vectors are rotated in the space (σ,τ,φ) in accordance with the permittivity tensor ε.

Then, we have that equation (32) is modified when we cross the plane to the new equation

$$\frac{1}{\varepsilon''}(\frac{\partial C'_0}{\partial\varphi})\frac{1}{\sinh\tau} = \varepsilon e^{ih(\omega_e)}\frac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta}\cos(\omega_0)D_0E_0 \quad (33)$$

Now, the unitary vector

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$$\hat{e}_\tau = \frac{1}{h_\tau}\frac{\partial \mathbf{r}}{\partial \tau} = \frac{(\cosh\tau - \cos\sigma)}{a}\frac{\partial \mathbf{r}}{\partial \tau}$$

$$= \frac{1}{\cosh\tau - \cos\sigma}\Big[(1-\cosh\tau\cos\sigma)\left(\cos\varphi\hat{e}_x + \sin\varphi\hat{e}_y\right)$$

$$-\sinh\tau\sin\sigma\hat{e}_z\Big] \quad (34)$$

Changes slightly only on the parameter, that is:

$$\underline{\hat{e}}_\tau = \frac{1}{h_\tau}\frac{\partial \mathbf{r}}{\partial \tau} = \frac{[\cosh\tau - \cos(\sigma+\gamma)]}{a}\frac{\partial \mathbf{r}}{\partial \tau}$$

$$-\sinh\tau\sin(\sigma+\gamma)\hat{e}_z\Big]$$

$$= \frac{1}{\cosh\tau - \cos(\sigma+\gamma)}\Big[[1-\cosh\tau\cos(\sigma+\gamma)]\left(\cos\varphi\hat{e}_x + \sin\varphi\hat{e}_y\right) \quad (35)$$

So, the angle between $\hat{e}_\tau$ the unitary basis vector $\underline{\hat{e}}_\tau$ for the incident component and the unitary basis vector for the scattered beam is

$$\cos\theta = \hat{e}_\tau \cdot \underline{\hat{e}}_\tau \quad (36)$$

We have also that the unitary vector

$$\hat{e}_\sigma = \frac{1}{h_\sigma}\frac{\partial \mathbf{r}}{\partial \sigma} = \frac{(\cosh\tau - \cos\sigma)}{a}\frac{\partial \mathbf{r}}{\partial \sigma}$$

$$= \frac{1}{\cosh\tau - \cos\sigma}\Big[-\sin\sigma\sinh\tau\left(\cos\varphi\hat{e}_x + \sin\varphi\hat{e}_y\right)$$

$$+\left(\cos\sigma\cosh\tau - 1\right)\hat{e}_z\Big] \quad (37)$$

Changes to a new form

$$\underline{\hat{e}}_\sigma = \frac{1}{h_\sigma}\frac{\partial \mathbf{r}}{\partial \sigma} = \frac{[\cosh\tau - \cos(\sigma+\gamma)]}{a}\frac{\partial \mathbf{r}}{\partial \sigma}$$

$$= \frac{1}{\cosh\tau - \cos(\sigma+\gamma)}\Big[-\sin(\sigma+\gamma)\sinh\tau\left(\cos\varphi\hat{e}_x + \sin\varphi\hat{e}_y\right)$$

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$$+[\cos(\sigma+\gamma)\cosh\tau-1]\hat{e}_z\Big] \qquad (38)$$

Again, the angle between the incident $\hat{e}_\sigma$ and the scattered $\hat{\underline{e}}_\sigma$ unitary basis vectors is now

$$\cos\psi=\hat{e}_\sigma\cdot\hat{\underline{e}}_\sigma \qquad (39)$$

The angles φ and ψ are not the angles between incident and refracted pulse respect the toroidal coordinates τ and σ but they are related to them, that is, while equation equations (32) and (33) are valid for after and before the plane $\varphi\equiv\Phi$, their left hand terms are calculated for values $\sigma=$ $\cos\beta\cos 2\varphi\sigma_{\varphi}+\cos\beta\sin 2\varphi\tau-\sin\beta\varphi_{\varphi}$ and $\varphi=$ $\sin\beta\cos\varphi\sigma_{.}+\sin\beta\sin 2\varphi\tau+\cos\beta\varphi$ and the right terms with the values $z\sigma_0$ and φ_0.

Now we remember that equation (32) is

$$\frac{1}{\varepsilon'}\left(\frac{\partial C_0}{\partial\varphi}\right)\frac{1}{\sinh\tau}=\varepsilon e^{ih(\omega_e)}\frac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta}\cos(\omega_0)D_0E_0 \qquad (40)$$

So, from equations (40) and (33) we see that the discontinuity in the derivative is then

$$\left(\frac{\partial C_0}{\partial\varphi}\right)_{\Phi+\varepsilon}-\left(\frac{\partial C_0}{\partial\varphi}\right)_{\Phi-\varepsilon}=(\varepsilon''-\varepsilon')(\sinh\tau)\varepsilon e^{ih(\omega_e)}\frac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta}\cos(\omega_0)D_0E_0 \qquad (41)$$

But for equation (33)

$$E'_\tau(\mathbf{r}_{1,2})=\frac{1}{\varepsilon'}\left(\frac{\partial C_0(\tau,\varphi)}{\partial\varphi}\right)\frac{1}{\sinh\tau}=\varepsilon e^{ih(\omega_e)}\frac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta}\cos(\omega_0)E_0 \qquad (29)$$

So that from (41) and (29) we obtain the slope:

$$\left[\left(\frac{\partial C_0}{\partial\varphi}\right)_{\Phi+\varepsilon}-\left(\frac{\partial C_0}{\partial\varphi}\right)_{\Phi-\varepsilon}\right]/E'_\tau=\frac{(\varepsilon''-\varepsilon')(\sinh\tau)\varepsilon e^{ih(\omega_e)}\frac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta}\cos(\omega_0)D_0E_0}{\varepsilon e^{ih(\omega_e)}\frac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta}\cos(\omega_0)E_0} \qquad (42)$$

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Or

$$[(\frac{\partial C_0}{\partial \varphi})_{\Phi+\varepsilon} - (\frac{\partial C_0}{\partial \varphi})_{\Phi-\varepsilon}] / E'_\tau = (\varepsilon'' - \varepsilon')(\sinh \tau) D_0 \quad (43)$$

From equation (43) we get that the angle of deflection is then

$$\Theta = \arctan[(\varepsilon'' - \varepsilon')(\sinh \tau) D_0] \quad (44)$$

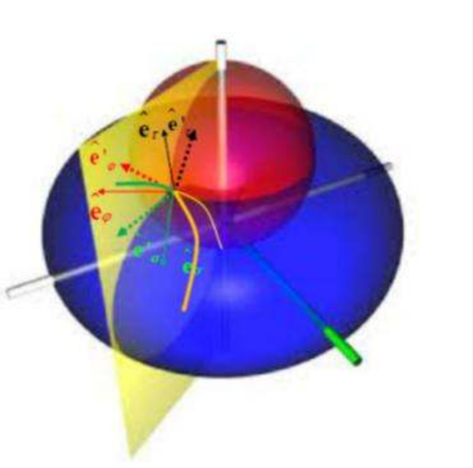
Assuming that the electric field only have a component, the total angle between the incident and refracted field will be

$$\Omega \equiv \theta + \Theta = \arccos \hat{e}_\tau \cdot \hat{\underline{e}}_\tau + \arctan[(\varepsilon'' - \varepsilon')(\sinh \tau) D_0] \quad (45)$$

The last equation requires an explanation, as the unit vectors change at every moment, we cannot suggest that this change ( ) has to do with the refractive index when it changes abruptly, but we must use the new unit vectors as a reference to measure the angle of deflection. This is where we measuretion Θ .

Figure 1

Scattering of a pulse by a plane on a torus



4 CONCLUSIONS

We have achieved the proposed goals of describing the deflection angle of an electromagnetic pulse traveling over a toroidal surface and suddenly reaching a region with an abrupt change in the permittivity tensor separating the donut into two parts.

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