

# Chapter 287

## Design and Simulation of a Model Reference Adaptive Control System Using the Recursive Least Squares Method with Forgetting Factor for Gain Adjustment



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### ABSTRACT

This article coupled the Recursive Least Squares Method with Forgetting Factor (RLS-FF) to a Model Reference Adaptive Control (MRAC) system and described an analysis for a second-order plant with variable and unknown parameters.

In the industrial context, manufacturing processes demand to be controlled, however there are variant and

even unknown parameters, a consequence of non-modeled dynamics. Thus, an algorithm capable of estimation the controller gains from the RLS-FF was proposed. Next, the MRAC simulation was carried out and the numerical results were obtained, regarding the target parameters of the control system. Through mathematical description and computational simulation, the results were promising, such as the convergence of controller gains. Therefore, this article aims to contribute with students and professionals in the field of Control and Automation, who are looking for models of adaptive control systems, in order to check, compare and implement new embedded technologies.

**Keywords:** Model Reference Adaptive Control (MRAC), Recursive Least Squares with Forgetting Factor (RLS-FF), Numerical simulation.

## 1 INTRODUCTION

The concept of adaptation is based on the principle of action whose objective is to change a certain behavior in order to adjust it under new conditions. In the context of industrial control, there are processes that are susceptible to changes in their dynamic behavior, either by unpredictable errors or by changes in their original operating parameters [1][2].

During the 1950s, the first studies directed to adaptive control design, which started to adjust its control action to adapt to dynamic changes in the process. In this context, it was during the development of autopilot systems for aircrafts that the first adaptive control models were developed [3]. Nevertheless, it was only during the 1970s and 1980s, when the mathematical arguments related to stability were properly proven, that the first commercial models of adaptive controllers appeared on the market [4].

Thus, the field of adaptive control research is significantly associated with the application of advanced mathematical tools. Thus, it becomes important the understanding of system identification methods, related to the field of mathematical modeling [5]. The use of dynamic system identification techniques allows modeling the process dynamics with little or no prior knowledge about the plant [6].

Among the system identification methods, it becomes possible to divide these into two distinct approaches: (i) offline methods, and (ii) online methods. The offline identification methods are methods based on an available set of samples [7]. On the other hand, an identification method is said to be online if it executes simultaneously the operation of the process to be identified, that is, it is updated recursively at each new sample [8]. In this sense, online methods have the essential characteristic of identifying parametric changes in online along the operation and control of the process [9].

One of the most widespread identification methods, with a detailed bibliography, is the Least Squares Method, an initially offline method, which can be modified to obtain a Recursive Least Squares with Forgetting Factor (RLS-FF) algorithm, which is an online recursive method. The RLS-FF is widely applied for parameter estimation in time-varying processes [6] [8].

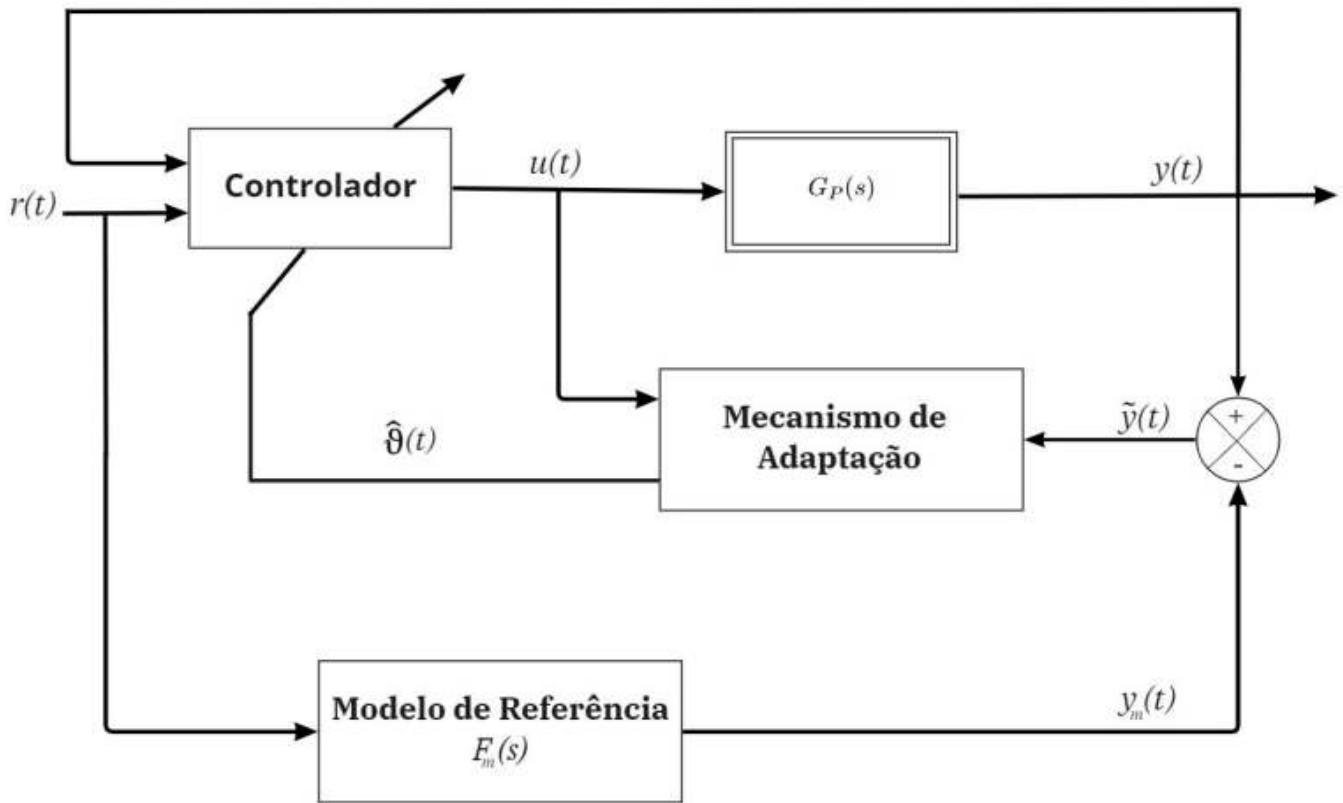
In this work, an application of an adaptive control design technique, called Model Reference Adaptive Control (MRAC). This technique essentially consists of a model-following system, thus establishing a model that describes the expected dynamic behavior of the closed-loop system. In this technique, an adaptation algorithm capable of updating the control law gains based on the difference between the process output and the model output was applied [11] [13].

This paper is organized as follows: Section II describes the Model Reference Adaptive Control (MRAC); Section III presents the fundamentals of the Least Squares Method; In Section IV, the modeling of the process to be controlled was demonstrated; in Section V, the methodology used in the work was established; in Section VI, the simulated results were described; and in Section VII, the results obtained were analyzed.

## **2 MODEL REFERENCE ADAPTIVE CONTROL (MRAC)**

Initially proposed in the late 1950s, the MRAC system, aims to regulate a given system so that its output signal tracks a certain signal obtained from the pre-established model in the design [10] [12] [14]. This reference signal is what characterizes this technique, and  $y_m(s)$  is the signal that describes the expected closed-loop behavior of the MRAC system. The block diagram of the typical MRAC system, is illustrated in Fig. 1 with the respective signal flows and processing blocks.

Fig. 1: Block diagram of Model Reference Adaptive Control



Source: Adapted from [12] [13].

The scheme illustrated in Fig. 1 establishes an adaptation mechanism as the central tool. This mechanism represents an algorithm that receives the tracking error information  $\tilde{y}(t)$ , which in turn computes the difference between the process output signal  $y(t)$  and the reference model output signal  $y_m(t)$ . This processing block simultaneously receives information from the control signal  $u(t)$ , which controls the process. From this information, the adaptation algorithm provides the system controller with a gain vector  $\hat{\theta}(t)$  [10] [11] [12]. For the construction of the adaptation mechanism, the insertion of a parameter estimation algorithm was performed.

Thus, an online identification method was adapted, this being from the measurements of the input and output signals of the process, which performed the online estimation of the plant parameters [11] [12]. Thus, in addition to performing parameter estimation of the dynamic system, there is a need to couple this algorithm to the adaptive control law [12] [13].

In the context of MRAC system design, the algorithm often used to compose the adaptation mechanism is the one developed by the Massachusetts Institute of Technology (MIT), the so-called Gradient Method (MG), also found in other bibliographies as the MIT Rule [10] [11] [12].

Thus, an ordinary first-order GP process was considered

$$G_P(s) = \frac{b_p}{s+a_p} \quad (1)$$

where  $G_p(s)$  is the open loop transfer function of the process,  $a_p$  and  $b_p$  are the plant parameters and  $s$  is the Laplace domain variable.

And a first-order reference model  $F_m$  of the form

$$F_m(s) = \frac{b_m}{s+a_m} \quad (2)$$

Being  $F_m(s)$  the transfer function of the reference model,  $a_m$  and  $b_m$  its respective parameters.

The process receives a control signal  $u(t)$  of the form

$$u(t) = \hat{\vartheta}^T(t)\omega(t) \quad (3)$$

With  $\omega(t)$  being a vector representing the regressors of the system, i.e., the signals that travel through the control loop so that  $\omega(t) = [y(t), r(t)]^T$ , with  $y(t)$  and  $r(t)$  the output and input signal streams, respectively, of the MRAC system and  $\hat{\vartheta}(t)$  represents a vector of estimated controller gains.

Then we applied the Laplace Transform to Eq.

$$U(s) = \hat{\vartheta}_1 R(s) - \hat{\vartheta}_2 Y(s) \quad (4)$$

The MIT Rule arises as a way to modify the parameters  $\hat{\vartheta}_1$  and  $\hat{\vartheta}_2$  in the negative gradient direction with the application of a cost function  $J(\vartheta)$ . This cost function is defined by

$$J(\vartheta) = \frac{1}{2} \tilde{y}^2 \rightarrow 0 \quad (5)$$

By the MIT rule, when you perform the partial derivative, you get

$$\frac{d\vartheta}{dt} = -\gamma \frac{\partial J}{\partial \vartheta} \quad (6)$$

where  $\gamma$  is a positive constant gain factor. We performed the partial derivative on both sides of the equality of Eq.

$$\frac{dJ}{d\vartheta} = \tilde{y} \frac{\partial \tilde{y}}{\partial \vartheta} \quad (7)$$

We therefore substituted Eq. (7) into (6) and obtained

$$\frac{dJ}{d\vartheta} = \tilde{y} \frac{\partial \tilde{y}}{\partial \vartheta} \quad (8)$$

For the plant and model outputs to be equivalent,  $Y(s) = Y_m(s)$

$$\frac{Y(s)}{R(s)} = \frac{Y_m(s)}{R(s)} \quad (9)$$

The result was compared to the reference model of Eq. (2)

$$\frac{b_p \vartheta_1}{s+a_p+b_p \vartheta_2} = \frac{b_m}{s+a_m} \quad (10)$$

However, Eq. (10) becomes true if both parameters  $a_p$  and  $b_p$  are known. However, when considering a MRAC design where the process parameters are unknown, a way around such a situation was sought.

Considering  $s$  as the representation of a time derivative, we have in the time domain

$$y(t) = \frac{b_p \hat{\vartheta}_1}{\frac{d(\cdot)}{dt} + a_p + b_p \hat{\vartheta}_2} r(t) \quad (11)$$

We performed the partial derivative of Eq. (11) with respect to  $\hat{\vartheta}_1$

$$\frac{\partial y(t)}{\partial \vartheta_1} = \frac{b_p}{\frac{d(\cdot)}{dt} + a_p + b_p \hat{\vartheta}_2} r(t) \quad (12)$$

As such, we partially derived Eq. (11) with respect to  $\hat{\vartheta}_2$

$$\frac{\partial y(t)}{\partial \vartheta_2} = \frac{b_p^2 \hat{\vartheta}_1}{\left(\frac{d(\cdot)}{dt} + a_p + b_p \hat{\vartheta}_2\right)^2} r(t) \quad (13)$$

The Eq. (13), can be manipulated to obtain

$$\frac{\partial y(t)}{\partial \vartheta_2} = - \frac{b_p \hat{\vartheta}_1}{\frac{d(\cdot)}{dt} + a_p + b_p \hat{\vartheta}_2} \left[ \frac{b_p}{\frac{d(\cdot)}{dt} + a_p + b_p \hat{\vartheta}_2} r(t) \right] \quad (14)$$

According to Eq. (12), we rearrange Eq. (14) as follows

$$\frac{\partial y(t)}{\partial \vartheta_2} = - \frac{b_p \hat{\vartheta}_1}{\frac{d(\cdot)}{dt} + a_p + b_p \hat{\vartheta}_2} y(t) \quad (15)$$

Thus we replaced the partial derivatives of Eqs. (12) and (15) in Eq. (8) in the form

$$\frac{\partial y(t)}{\partial \vartheta_2} = - \frac{b_P \hat{\vartheta}_1}{\frac{d(\cdot)}{dt} + a_P + b_P \hat{\vartheta}_2} y(t) \quad (16)$$

And for  $\hat{\vartheta}_2$

$$\frac{d\hat{\vartheta}_2}{dt} = \gamma \tilde{y} \frac{b_P \hat{\vartheta}_1}{\frac{d(\cdot)}{dt} + a_P + b_P \hat{\vartheta}_2} y(t) \quad (17)$$

Since  $a_p$  and  $b_p$  are unknown, the following approximation was made

$$a_P + b_P \hat{\vartheta}_2 \cong a_m \quad (18)$$

Therefore, one can rewrite Eq. (16) and (17) with parameters from the reference model only, as follows

$$\frac{d\hat{\vartheta}_1}{dt} = -\gamma \tilde{y} \left[ \frac{a_m}{\frac{d(\cdot)}{dt} + a_m} r(t) \right] \quad (19)$$

And for  $\hat{\vartheta}_2$

$$\frac{d\hat{\vartheta}_2}{dt} = \gamma \tilde{y} \left[ \frac{a_m}{\frac{d(\cdot)}{dt} + a_m} y(t) \right] \quad (20)$$

Being  $\frac{a_m}{s+a_m}$  represents a filter  $\phi(t)$  applied on the regressor signals.

For processes of order  $n \geq 1$ , the auxiliary filter can be run in the form

$$\phi(t) = F_m(s) \mathbf{I} \omega \quad (21)$$

The evaluation of the estimation error, or augmented error, was also considered. This is fundamental for stability analysis, so the following equality was considered

$$F_m(s) (\vartheta^{*T} \omega) = \vartheta^{*T} (F_m(s) \omega) \quad (22)$$

We rewrote Eq. (22) as

$$F_m(s) (\vartheta^T \omega) - F_m(s) (\hat{\vartheta}^T \omega) = \vartheta^T (F_m(s) \omega) - \hat{\vartheta}^T (F_m(s) \omega) \quad (23)$$

The Eq. (21) was considered so that Eq. (23) was rewritten in the form

$$\hat{\vartheta}^T \phi - F_m(s)(\hat{\vartheta}^T \omega) = \vartheta^T \phi - F_m(s)(\vartheta^T \omega) \quad (24)$$

The equality of Eq. (24) favors the calculation of the augmented error in a digital environment, as this represents the accumulated error of the digital estimation of the desired parameter. Thus, we added the term  $\hat{\vartheta}^T \phi - F_m(s)(\hat{\vartheta}^T \omega)$  to the tracking error value  $\tilde{y}[k]$ , to digitally obtain the value of the estimation error

$$E_1[k] = y[k] + \hat{\vartheta}^T [k]\phi[k] - F_m(s)(\hat{\vartheta}^T [k]\omega[k]) \quad (25)$$

With this, the following adaptive law was used for processes of order  $n \geq 1$  to update the parameter vector  $\vartheta$  of the normalized form [11]

$$\frac{d\hat{\vartheta}}{dt} = \gamma \frac{\varphi(t)E_1(t)}{1+\varphi(t)\varphi^T(t)} \quad (26)$$

Where  $[1 + \varphi(t)\varphi^T(t)]$  is a normalization sign to ensure that the derivative is always negative [12].

### 3 RECURSIVE LEAST SQUARES METHOD WITH FORGETTING FACTOR (RLS -FE)

To model any control system it is essential to have some information about the dynamics of the process to be controlled. In this sense, processes with varying dynamic characteristics, called time-varying systems, offer a challenge to modeling the control system. In these cases, it becomes essential that the estimation of the system parameters weights more heavily the most recent information of its dynamics, thus a recursive estimator is recommended [6][9].

Thus, the Recursive Least Squares (RLS) method can be applied to estimate the plant parameters in an alternative way to the MIT Rule [12] [13]. Similar to the MIT Rule, the RLS algorithm aims to minimize the quadratic error cost function, expressed by

$$J(\hat{\vartheta}) = \frac{1}{2} \int_0^t E_1^2(\tau) d\tau \quad (27)$$

An RLS type parameter estimation law is characterized by having a covariance matrix  $P$ , which stores the direction information of the parameters to be estimated [8][12]. The  $P$  matrix can be used to change the speed of adaptation of the controller gains as a function of time or samples, and is updated as

$$\frac{dP}{dt} = - \frac{P(t)\phi(t)\phi^T(t)P(t)}{1+\phi(t)\phi^T(t)} \quad (28)$$

With the covariance matrix computation, the gain vector  $\hat{\theta}$  is updated in the form [8][12].

$$\frac{d\hat{\theta}}{dt} = -P(t) \frac{\phi(t)E_1(t)}{1+\phi(t)\phi^T(t)} \quad (29)$$

where  $\phi(t)$  and  $E_1(t)$  are updated by means of the Eqs. (21) and (25) respectively.

For a digital implementation, a discretization method can be applied in the form of obtaining a sample ratio for both time derivatives of Eqs. (28) and (29). An algorithm for computing the discrete  $P$  covariance matrix by the Bilinear Approximation is expressed by

$$P[k] = P[k-1] - \frac{T}{2} \frac{P[k]\phi[k]\phi^T[k]P[k]}{1+\phi[k]\phi^T[k]} \dots$$

$$- \frac{T}{2} \frac{P[k-1]\phi[k-1]\phi^T[k-1]P[k-1]}{1+\phi[k-1]\phi^T[k-1]} \quad (30)$$

Similarly,  $\hat{\theta}$  can be updated according to

$$\hat{\theta}[k] = \hat{\theta}[k-1] - \frac{T}{2} P[k] \frac{\phi[k]E_1[k]}{1+\phi[k]\phi^T[k]} \dots$$

$$- \frac{T}{2} P[k-1] \frac{\phi[k-1]E_1[k-1]}{1+\phi[k-1]\phi^T[k-1]} \quad (31)$$

For the algorithm to update the parameter variations with greater capacity, the digital RLS algorithm can be rewritten by applying a forgetting factor, the so-called Recursive Least Squares with Forgetting Factor (RLS -FF) algorithm [6].

$$P[k] = \frac{1}{\lambda} P[k-1] - \frac{1}{\lambda} \left\{ \frac{T}{2} \frac{P[k]\phi[k]\phi^T[k]P[k]}{\lambda + \phi[k]\phi^T[k]} \right\} \dots$$

$$- \frac{1}{\lambda} \left\{ \frac{T}{2} \frac{P[k-1]\phi[k-1]\phi^T[k-1]P[k-1]}{\lambda + \phi[k-1]\phi^T[k-1]} \right\} \quad (32)$$

Eq. (31) can be rewritten in the form

$$\hat{\theta}[k] = \hat{\theta}[k-1] - \frac{T}{2} P[k] \frac{\phi[k]E_1[k]}{\lambda + \phi[k]\phi^T[k]} \dots$$

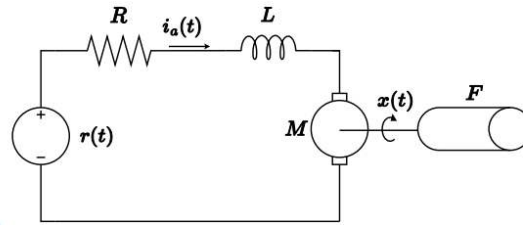
$$- \frac{T}{2} P[k-1] \frac{\phi[k-1]E_1[k-1]}{\lambda + \phi[k-1]\phi^T[k-1]} \quad (33)$$

Where  $\lambda$  is the forgetting factor,  $\lambda \in (0,1)$ .

#### 4 PROCESS MODELING

To validate the concepts developed in Section II and III, a second-order plant with time-varying parameters was selected. The selected plant was a state-space model of a direct current (DC) servo motor depicted in Fig. 2.

Fig. 2: Equivalent circuit for the DC motor model



Source: Adapted from [15].

As per the circuit in Fig. 2, the servopositioning process was modeled based on the electrical parameters of resistance  $R$ , supply voltage  $r(t)$ , armature current  $i_a(t)$  and inductance  $L$ . The model also presents mechanical and electromechanical parameters, such as the rotor itself  $M$  and the values of mechanical energy  $F$  and angular speed  $x(t)$  [15].

According to the circuit, Fig. 2, the equation of angular motion can be described as

$$\frac{dx}{dt} = -bx(t) + T_F \quad (34)$$

where  $T_F = k_f i_a$ ,  $k_f$  is the force proportionality constant,  $b$  the torque in  $Nm$ .

The Eq. (34) can be rewritten as

$$\frac{dx}{dt} = -\frac{b}{F}x(t) + \frac{k_f}{F}i_a \quad (35)$$

Thus, the summation of the circuit voltages was analyzed in order to obtain the following loop equation

$$L \frac{di_a}{dt} + Ri_a + V_M - V = 0 \quad (36)$$

Where  $V_M = k_e x(t)$  is the rotor's electromotive voltage.

We rewrote Eq. (36) as a function of the derivative of the motor current, as follows

$$\frac{di_a}{dt} = -\frac{R}{L}i_a(t) - \frac{k_e}{L}x(t) + \frac{1}{L}r(t) \quad (37)$$

Thus, with the differential equations obtained in (35) and (37), the following state-space model is obtained

$$\begin{bmatrix} \dot{x}(t) \\ \dot{i}_a(t) \end{bmatrix} = \begin{bmatrix} -b/F & k_f/F \\ -k_e/L & -R/L \end{bmatrix} \begin{bmatrix} x(t) \\ i_a(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} r(t) \quad (38)$$

$$y(t) = [1 \quad 0] \begin{bmatrix} x(t) \\ i_a(t) \end{bmatrix} \quad (39)$$

consider  $k_f = k_e = k$ .

The model described by Eq. (38) and (39) will be used to validate the designed MRAC based on the RLS-FF algorithm

## 5 METHODOLOGY

As described in Section IV, a servo positioning process of a DC motor was used. Thus, based on the state equations presented, the following parameters were selected for simulation of the process.

$$\begin{bmatrix} \dot{x}(t) \\ \dot{i}_a(t) \end{bmatrix} = \begin{bmatrix} -\frac{0,1 \text{ kg.m}^2/\text{s}}{0,01 \text{ kg.m}^2} & \frac{0,01 \text{ Nm/A}}{0,01 \text{ kg.m}^2} \\ -\frac{0,01 \text{ Nm/A}}{0,5 \text{ H}} & -\frac{R}{0,5 \text{ H}} \end{bmatrix} \begin{bmatrix} x(t) \\ i_a(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{0,5 \text{ H}} \end{bmatrix} r(t) \quad (40)$$

The resistance parameter  $R$ , in turn, can vary with changes in temperature or wear over time, so we considered  $R$  as a time-varying parameter of the form

$$R = \begin{cases} 1 \Omega, & 0 \leq t < 120 \\ 0,92 \Omega, & t \geq 120 \end{cases} \quad (41)$$

For the validation of the MRAC based on the RLS-FF algorithm, no knowledge of the parameters arbitrated here is required, only of the process order. Prior knowledge of the order enables the correct selection of the reference model for an adequate response in the simulation. Thus, a reference model in the Laplace domain was selected to obtain the following design specification.

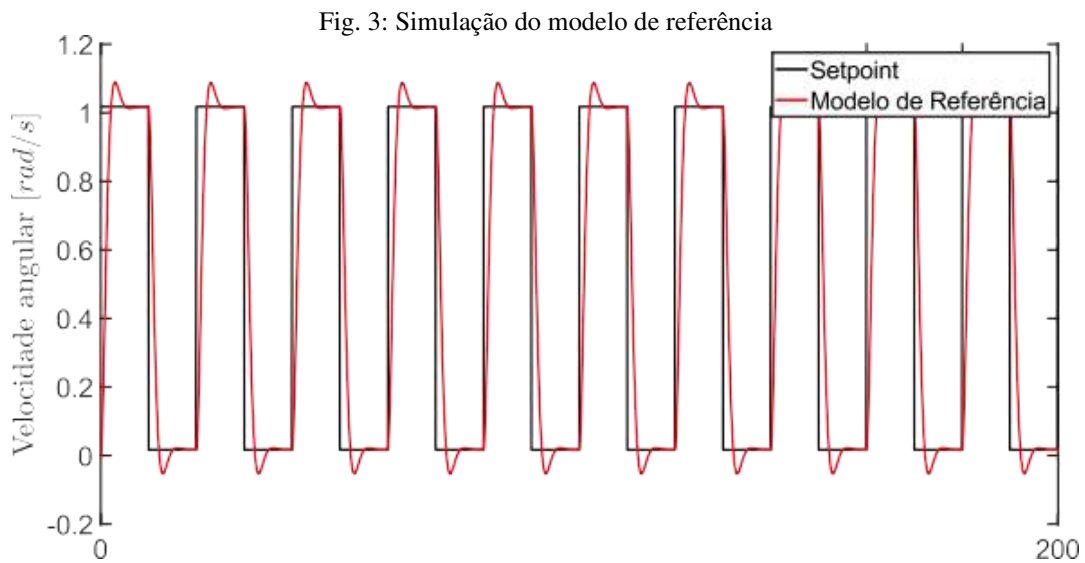
$$F_m(s) = \begin{cases} T_p = 3 \\ M_p = 0,07 \end{cases} \quad (42)$$

Being  $T_p$  the peak time in seconds and  $M_p$  being the maximum spare signal of the response.

Thus, the reference model  $F_m(s)$  can be defined as a transfer function of the form

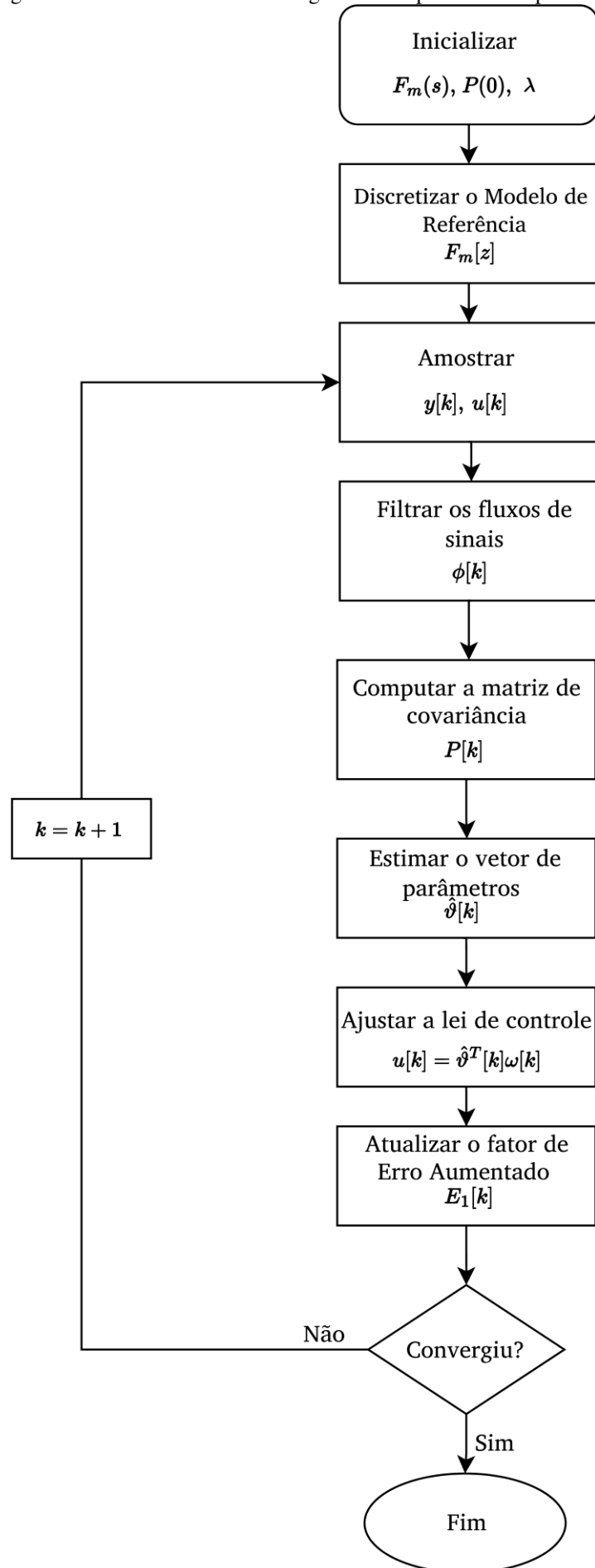
$$F_m(s) = \frac{1,882}{s^2 + 1,773s + 1,882} \quad (43)$$

In Fig. 3, the simulation of the reference model for a positive pulsed signal input is illustrated.



In Fig. 4, a flowchart of the operation of the RLS-FF algorithm coupled with the adaptive law of the MRAC system is depicted.

Fig. 4: Flowchart for the RLS-FF algorithm coupled with adaptive law



Therefore, to validate the controller design and the proposed RLS-FF algorithm, the proposed DC motor model and the selected reference model will be initially simulated.

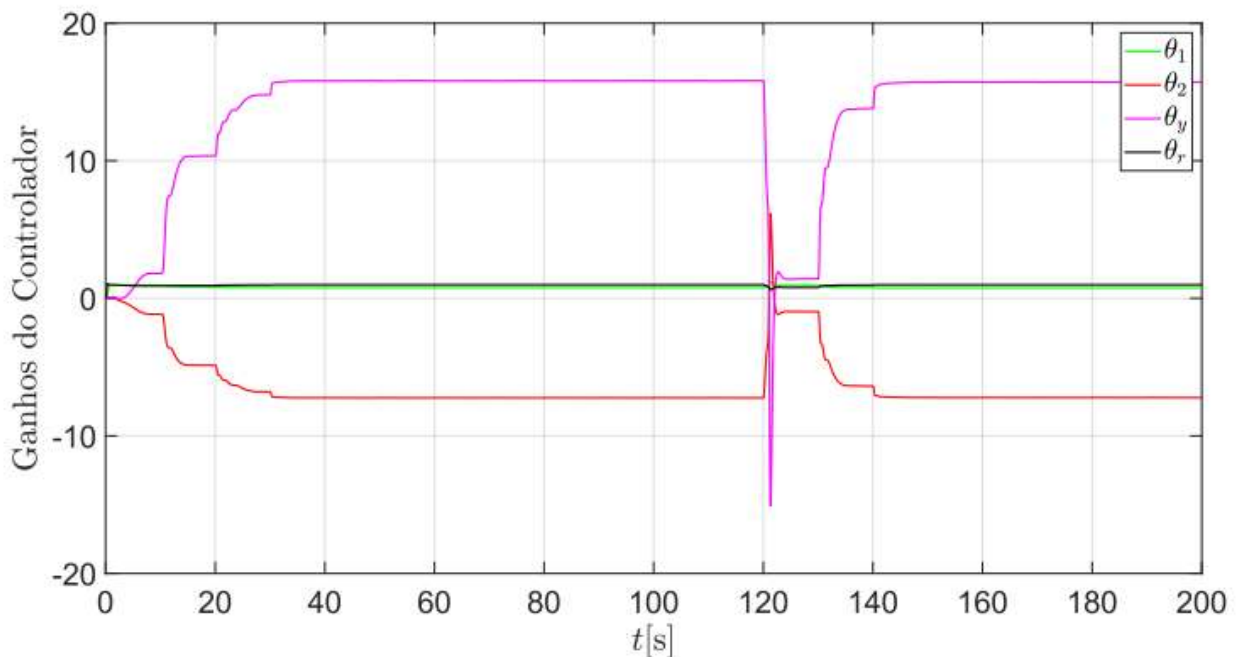
Next, the RLS-FF algorithm proposed in Section III will be executed. Both the progression of the parameters towards the gains required for the process output to follow the established model, as well as the tracking error rate and the control action signal provided by the main controller of the MRAC system should be evaluated.

## 6 RESULTS

For the design of the MRAC based on the adaptive law of RLS-FF, represented already in discrete form by Eqs. (33) and (32). For the computation of the covariance matrix  $P$  the initialization of this matrix of the form  $P(0) = P_0 I$  with  $P_0 = 10,000$  was done. Note that it is important to initialize this matrix with a significantly high value so that the controller gain estimation algorithm is fast enough to follow the performance specifications, this value can be adjusted as needed for performance.

From a routine of runs and tests, a forgetting factor  $\lambda = 0,9958$  was selected. This value was set so that the controller would accurately adjust to the parametric change of  $R$  during its operation. For the entire simulation, a constant sampling rate of  $T = 0,01 s$  was used. In Fig. 5 the progression of the controller parameters is illustrated. A large variation can be seen at  $t = 120 s$ , where a change in the rotor resistance value occurs.

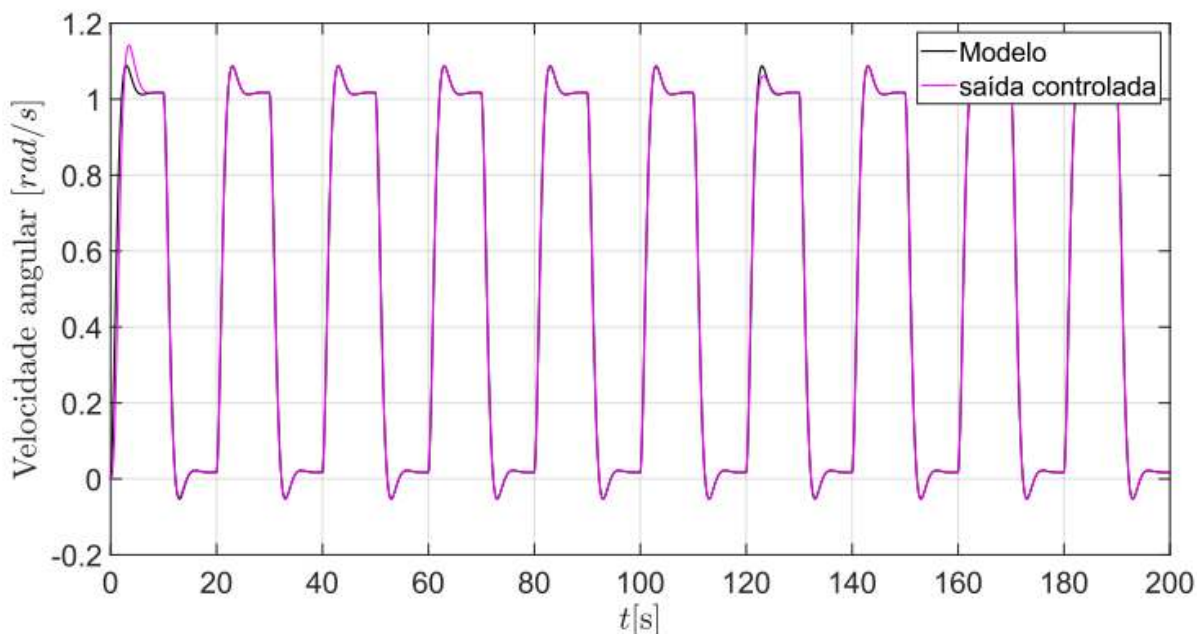
Fig. 5: Estimation of the controller gains



One can also notice the efficiency of the algorithm in storing information about the direction of the parameters. This occurs due to the update of the covariance matrix at each iteration.

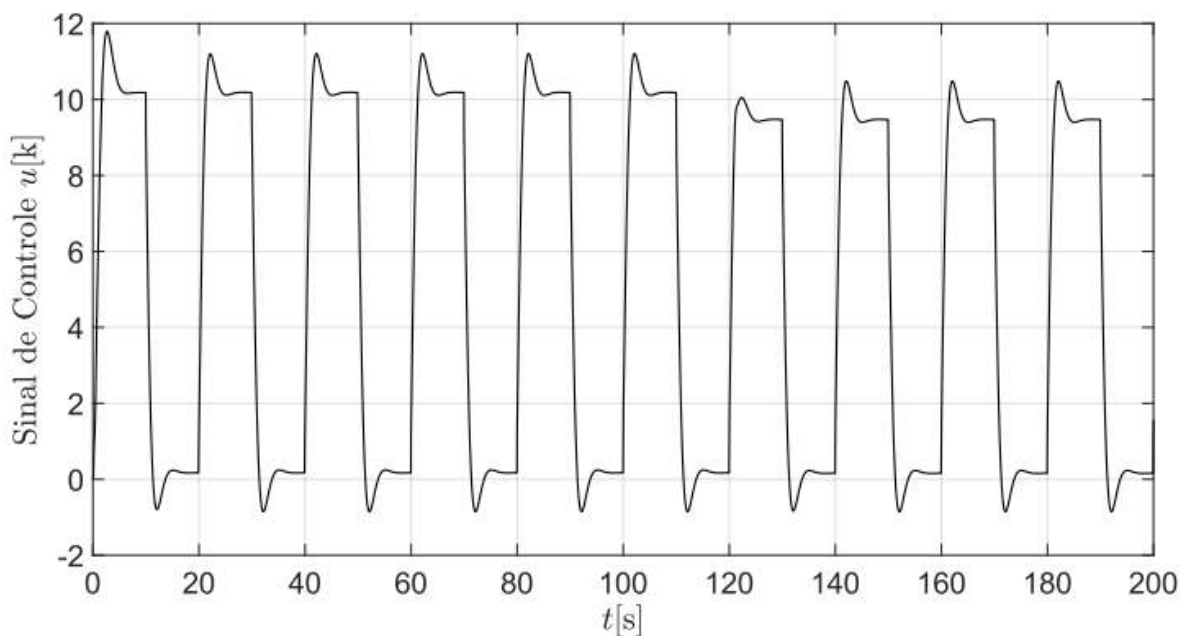
To validate the designed MRAC, Fig. 6 represents the output of the controlled system with respect to the reference model defined in Eq. (43).

Fig. 6: Controlled output according to the RLS -FF algorithm



The performance of the MRAC can also be analyzed according to the control action signal  $u[k]$  generated by the controller in open loop with the plant. This control action is represented in Fig. 7.

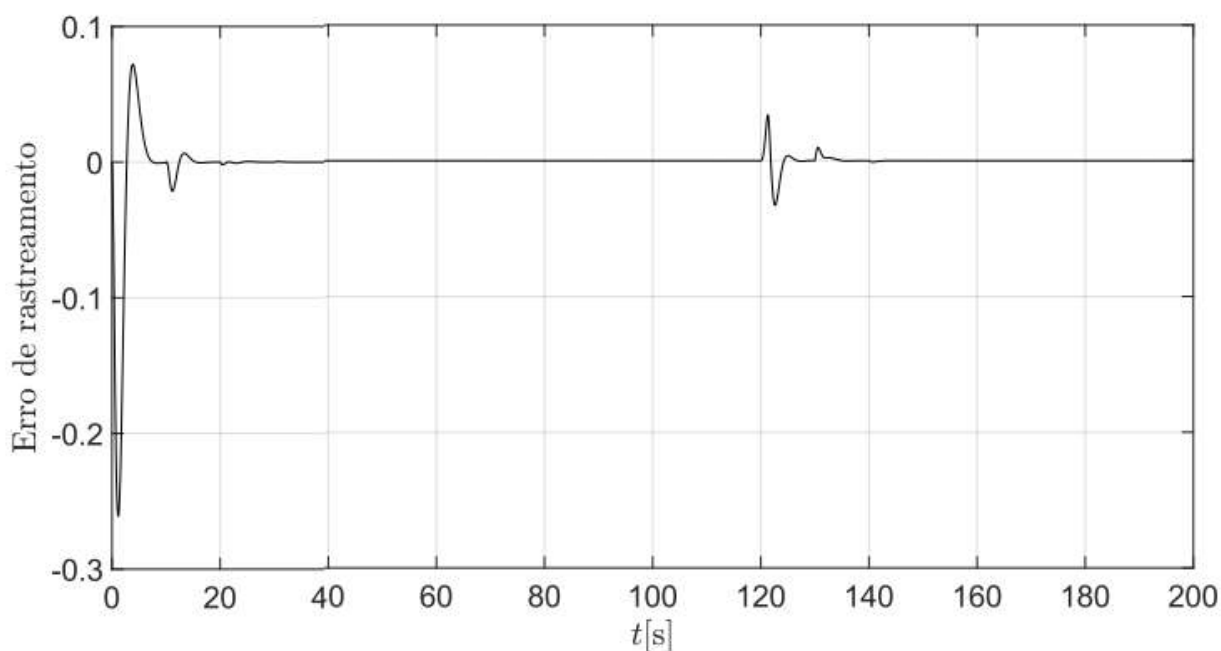
Fig. 7: Control action for the gains estimated by the RLS -FF algorithm



Finally, Fig. 8 shows the tracking error signal. That is, how much the process output deviated from the reference model output, which represents the desired design specifications. This is an important

performance criterion for the closed loop system, because it quantitatively shows if the controller has significantly tracked the model output, which contains the pre-set specifications.

Fig. 8: Erro de rastreamento do sistema CAMR



This is mainly due to the adaptation algorithm being a recursive algorithm, i.e. the more recent samples available, the more efficient the RLS-FF algorithm becomes in estimating the necessary parameters for the controller.

## 7 CONCLUSION

In this paper, RLS-FF was coupled with the MRAC system. Thus, we used the RLS-FF as a replacement for the typical GM used in the design of the Model Reference Adaptive Control.

Thus, a second-order process with variant parameters was proposed, which was simulated in MATLAB® software and, as a consequence, the analysis of the RLS-FF coupling to the MRAC system in discrete time was performed.

The results validated the proposed algorithm for cases where the parameters of the plant to be controlled are not completely known. The proposed control system, maintained performance even with parametric variations that adversely affect the performance of the control system.

In this sense, the MRAC system designed from RLS-FF for the adaptation law offered a promising result for processes of dynamic characteristics with varying or even unknown parameters. It demonstrated numerical capability and convergence of parameter values, as well as maintained the output of the control system as per the pre-established performance specifications.

Through this work, the authors believe in motivating other professionals to explore the issues addressed here, as well as providing bibliographic material for study, checking and comparison of students and other professionals in the area of industrial control and automation.

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