


# Chapter 79

## Mental calculation strategies used by teachers in the additive field: from mathematical knowledge to methodological implications for teaching

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### ABSTRACT

The new curricular guidelines highlight the importance of the development of mental calculus skills and their relationship with the objects of knowledge for the early years of elementary school. This paper presents the clipping of a research developed by Brazilian and Portuguese researchers. We present the strategies and understanding of mental calculus used in the additive field of whole numbers up to 100 per teacher and work in the public network of a municipality located in the northwest region of the State of Paraná, in the initial years of elementary school. Twenty-six teachers who solved mentally and in two different ways participated in this project, six propositions that involved the addition and subtraction with numbers of two digits of whole numbers up to 100. The analysis of the results showed that this group used the algorithmic procedures mentally and the strategies 1010 and N10, in addition to the commutative, and associative properties of the addition and the distributive multiplication of the addition. Subsequently, we interviewed to identify

what mathematical knowledge was underlying the resolutions undertaken. It is observed that although they use the characteristics of the Decimal Numbering System (NdS), even in algorithmic resolution, the properties of operations and strategies already presented in studies on the subject were not identified by the participants about the mental calculus undertaken, and the mathematical knowledge underlying and Le. These data reveal the little knowledge of teachers about the characteristics of ndS and the properties of operations that allow us to flexible mental calculation strategies and their variations according to the numbers involved in the operations, evidencing the need for teaching training on the subject investigated. This cutout of the researched data brings significant aspects to be considered in a process of continuous training of teachers and teachers who teach mathematics as a contribution to this process.

**Keywords:** Common National Curriculum Base, Mathematics Education, Elementary School, Mental calculation strategies, Continuing Education, Teacher training.

### 1 INTRODUCTION

This work aims to present the mental calculation strategies used in the additive field of whole numbers up to 100 per teacher and working in the public network of a municipality located in the northwest region of the State of Paraná, in the initial years of elementary school, as well as the understanding shown by them about these procedures. This section comes from research developed in <sup>1</sup> collaboration with

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<sup>1</sup> The project submitted and approved by the Ethics Committee under no. CAAE 42616821.0.0000.0104 in which they participated, in addition to the author of this text, the following Members: Pedro da Cruz Almeida, Graciosa Veloso, Regina Maria Pavanello, Ademir Pereira Jr., Lincoln Naranti dos Santos, Lucilene Lusía Adorno de Oliveira, and Sandra Regina D'Antonio Verrengy.

professor and members of the Disciplinary Center for Educational Studies (CIED), of the Higher School of Education of the Polytechnic Institute of Lisbon (ESELx), whose objective was to investigate the mental calculation strategies used by Brazilian teachers working in the public network of a municipality located in the northwest region of the State of Paraná and The Portuguese students are part of the initial training course for the teaching of the 1st and 2nd cycles of the Higher School of Education of the Polytechnic Institute of Lisbon (ESELx), in the additive field of whole numbers up to 100, intending to base the strategies that support the procedures used and the importance for the teaching and learning process in understanding the operations and their properties.

Regarding calculations, we considered what is put in the National Common Curriculum Base (BNCC) (BRASIL, 2017), which establishes that students in the initial years of elementary school

[...] develop different strategies for obtaining the results, especially by estimation and mental calculation, in addition to algorithms and use of calculators that are included in curricular proposals in Brazil, without, however, explaining or being a topic developed in the teacher education of future professionals who will work in the initial years (BRASIL, 2017, p. 268).

In this document, mental calculus is related to both the object of knowledge and strategy and procedure, which can be observed in Table 1:

Table 1: Mental calculation ability and its relationship to the objects of knowledge at BNCC for the early years of elementary school

<b>OBJECT OF KNOWLEDGE</b>	<b>SKILLS</b>
Construction of fundamental facts of addition and subtraction	(EF02MA05) Build basic facts of addition and subtraction and use them in mental or written calculus.
Construction of fundamental facts of addition, subtraction, and multiplication.	(EF03MA03) Build and utilize basic addition and multiplication facts for mental or written calculus. (EF03MA04) Establish the relationship between natural numbers and points of the numerical line to use it in the ordering of natural numbers and also in the construction of addition and subtraction facts, relating them with shifts to the right or left.
Calculation procedures (mental and written) with natural numbers: addition and subtraction.	(EF03MA05) Use different mental and written calculation procedures, including conventional ones, to solve significant problems involving addition and subtraction with natural numbers.
Problems involving meanings of addition and subtraction: joining, adding, separating, removing, comparing, and completing quantities	(EF03MA06) Solve and elaborate problems of addition and subtraction with the meanings of joining, adding, separating, removing, comparing, and completing quantities, using different exact or approximate calculation strategies, including mental calculation.
Properties of operations for the development of different calculation strategies with natural numbers	(EF04MA03) Solve and elaborate problems with natural numbers involving addition and subtraction, using various strategies, such as calculation, mental calculation, and algorithms, in addition to making estimates of the result. (EF04MA04) Use the relationships between addition and subtraction, as well as between multiplication and division, to expand calculation strategies. (EF04MA05) Use the properties of operations to develop calculation strategies.
Problems involving different meanings of multiplication and division: addition of equal plots, a rectangular configuration,	(EF04MA06) Solve and elaborate problems involving different meanings of multiplication (addition of equal plots, rectangular organization, and proportionality), using various strategies, such as calculation by estimation, mental calculation, and algorithms.

Methodology focused on the area of interdisciplinarity:

*Mental calculation strategies used by teachers in the additive field: from mathematical knowledge to methodological implications for teaching*

proportionality, equitable distribution, and measure	(EF04MA07) Solve and elaborate division problems whose divider has a maximum of two digits, involving the meanings of equitable distribution and measurement, using various strategies, such as calculation by estimation, mental calculation, and algorithms.
Counting issues	(EF04MA08) Solve, with image support and/or manipulated material, simple counting problems, such as determining the number of possible groupings by combining each element of one collection with all elements of another, using strategies and forms of personal registration.
Problems: addition and subtraction of natural numbers and rational numbers whose decimal representation is finite.	(EF05MA08) Solve and elaborate problems of multiplication and division with natural numbers and with rational numbers whose decimal representation is finite (with natural multiplier and natural divisor and different from zero), using various strategies, such as calculation by estimation, mental calculation, and algorithms.

Source: Common National Curriculum Base (BRASIL, 2017, p. 282-295)

However, despite being mentioned in the numbers thematic unit, the BNCC does not provide indications of how to develop mental calculus and it was observed that in Brazil, the ability to numerically calculate is related to the competence to solve operations algorithmically, which is the focus of teaching in the early years of an elementary school in this thematic unit (DA COSTA; PAVANELLO, 2017) as opposed to mental calculus.

Studies conducted by Pais e Freitas (2015, p. 114) note "the persistent lack of attention given to the teaching of mental calculus, [...] and that by the end of the 20th century, it had not yet been treated sufficiently." Alfonso (2005, p. 28) observed that teachers are reticent about teaching calculus for several reasons, including the beliefs that this teaching is an obstacle to learning general methods, a waste of time because the calculator can solve the teacher's difficulties in performing them and failing in front of students, the massification of the class, pressure on the contents, lack of time, absence of treatment given to mental calculus in textbooks, emphasis on operative technique, and lack of well-founded teaching materials for the teaching of mental calculus.

Thus, this text presents the mental calculation strategies used in the additive field of whole numbers up to 100 per teacher and worked in the public network of a municipality located in the northwest region of the State of Paraná, in the initial years of elementary school, as well as the understanding shown by them about these procedures.

## 2 THEORETICAL FRAMEWORK

As observed by Da Costa and Pavanello (2017), before BNCC (BRAZIL, 2017), the National Curricular Parameters (PCNs) (BRASIL, 1997, p. 74) established that a good calculus skill depends on several factors, including the mastery of certain arithmetic processes known as table, lists of fundamental facts, laws, etc., a basic repertoire that allows students to perceive, even if intuitively, properties of operations, including associativity, commuting in addition, multiplication and some regularities, such as all results being paired in multiplications by 2, the results end in zero or five, in the table of five, among others.

And stand document also observes that the construction of this basic repertoire is what will support the expansion of the different types of calculation (mental or written, accurate or approximate) complementary and related, since "the written calculation, to be understood, is based on mental calculation and estimates and approximations" (BRASIL, 1997, p. 75).

The mental calculation explores the properties of numbers and operations (DOUADY, 1994) and is a set of procedures that articulate to obtain an accurate or approximate result, without resorting to a pre-established algorithm and relying on the properties of the numeral numbering system (SND) (PARRA, 1996, p. 189). Other strategies are regrouping around double the number, for example,  $7 + 9 = 7 + 7 + 2$ , or in clusters of 10, as in the case of  $8 + 5 = (8 + 2) + 3$ , or even in regrouping around the 5 ( $5 + 5$ ) + 3 (KAMII, 1984, 1990). Another resource used for mental calculation is the use of the line, whether numbered or not, which, in general, is part of the decomposition of the numbers, associating them to form groups of 10.

Mental calculation is important for developing the meaning of numbers or numerical sense because they flexibly use numbers (ONION, 2002) since it is necessary to "evaluate the magnitude of the numbers and judge the reasonableness of the results; know how to use different representations of numbers; relate numbers, symbols, and operations and use them to understand numerical situations" (REYS; 1998 apud ONION, 2002, p. 231).

Mental calculus has two characteristics: the fact that the numbers are treated holistically and the calculations that, seem similar, can use different strategies depending on the numbers involved (GREAT BRITAIN, 1999, p. 14). Sensitizing students to the development of more efficient methods of calculating provides greater confidence in their operative capacity (GREAT BRITAIN, 1999, p. 14), which can be developed when socializing the answers, explaining the reasoning strategies used in the confrontation with the group, verifying in this context the efficiency of each of them.

The guidelines on the curriculum for English primary teachers (GREAT BRITAIN, 1999) indicate that for the development of these processes, the teacher must identify the strategies used by students, draw attention to those who are different, and suggest to the most efficient students, in an environment in which students feel comfortable to expose their strategies and listen carefully to their colleagues' explanations, comparing them and discussing them (THOMPSON, 2009, p. 41), also enabling the student to regulate their learning and taking "[...] awareness about and have greater control of the learning process it is performing" (BILHALBA, 2019, p. 36).

Regarding the strategies possible to be used, Hartinett (2007) surveys the studies conducted by different authors and describes some of them for addition and mental subtraction: the so-called 1010, when the numbers are divided into tens and units and work with the parts separately, from left to right and N10, in which a number is divided into tens and units and tens of the second number are added to the first number, such as:  $(28+35=; 20+30=50; 5+8=13; 50+13=63)$ .

It also describes A10, in which the second number is divided to facilitate a bridge to a multiple of ten, and then the rest is added to the first number and the N10C in which the second number is rounded to a multiple of ten, and this number is added to the first number followed by an adjustment or compensation by rounding.

There are also other strategies and guidelines for teaching mental calculation strategies, which we will address at another time, but these data show different possibilities to calculate mentally and contribute to the flexibilization of thought, important aspects in the development of skills for the appropriation of knowledge objects proposed by BNCC (BRASIL, 2017).

### 3 RESEARCH METHODOLOGY

The research was mostly developed during the period of the health crisis due to COVID-19, we used technological instruments and tools, available free of charge to all those involved in all stages of the research<sup>2</sup>.

In this process, a questionnaire was applied with data for a characterization of the participants, as well as six calculation propositions that involved the addition and subtraction with numbers of two digits of integers up to 100 for the participants to solve them in two different ways: for the addition ( $47 + 23$ ;  $53 + 38$  and  $\_\_\_\_\_ + \_\_\_\_\_ = 78$ ) and subtraction ( $58 - 35$ ;  $98 - 49$  and  $87 - \_\_\_\_\_ = 58$ ).

Subsequently, we analyzed the mental calculation strategies used by teachers, based on the categorization evidenced by Hartinett (2007) and described above; we selected some of them for discussion with them; we elaborated some tasks to be developed with their students and discussed the results of these applications.

The cutout showed the mental calculation strategies used by teachers who work in the early years of elementary school and the mathematical knowledge underlying these resolutions, as well as the understanding is shown by them about these aspects.

### 4 DESCRIPTION AND DATA ANALYSIS

Twenty-six teachers who work in the early years of the elementary school participated in this project, of which 15 *had lato sensu* specialization, in addition to graduation, and 11 completed or attended master's degree. Regarding the professional experience of teachers in the initial years of elementary school, most of the participants worked or worked in the first three initial years of elementary school and only 3 of them worked in the 4th year and 2 in the 5th grade.

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<sup>2</sup> COVID-19 is an infectious disease caused by coronavirus SARS-CoV-2. On January 30, 2020, they declared that the outbreak of the new coronavirus constitutes a Public Health Emergency of International Importance (ESPII) – the highest level of alert of the Organization, as provided for in the International Health Regulations.

Regarding the formative experience of mental calculation strategies, it was observed that most teachers did not have the training to develop this theme and only three of them reported having had this training in teaching training courses<sup>3</sup>.

The collected data confirm what motivated this research, that is, that mental calculus is not part of the teaching education of our teachers and is eventually approached in a teaching training course.

Another aspect that drew attention is the fact that most teachers work in the first three years of elementary school, whose theme should be widely explored, but did not obtain formative experiences for this. In this sense, it can also be observed that this training is independent of the period in which it was performed, since the age range of the research participants included both recent graduates and those who have graduated longer.

For the choice of operations, we considered the multiple possibilities of operating numerically with the numbers from the different strategic possibilities for this resolution. Regarding the resolution of the proposed addition and subtraction operations, they were asked to present two different processes.

The analysis of the results of the addition operations made it possible to realize that the group uses algorithmic procedures for resolution and called the fact of reaching a result without the use of pencil and paper, which led us to infer that they did so "head-on", or mentally and this is the procedure that prevails, for them, in the resolution of operations.

In addition to operations, the strategies involved the decimal decomposition of the two terms (1010), possibly because the SND has as characteristic the positional value and is additive. In addition to the characteristics of the NdS, some teachers used the commutative and associative properties of the addition that this strategy allows.

In subtraction operations, in addition to algorithmic procedures, some of the resolutions presented the decimal decomposition of one of the terms (N10). In this case, it was also considered that the SND has as characteristic the positional value and the additive being and also the distributive property of multiplication concerning the addition when using strategy 1010.

In the interview, it was investigated how mathematical knowledge was underlying the resolutions undertaken by the participants and we observed that although they used the characteristics of the NDS, even in algorithmic resolution, the properties of the operations and strategies already presented in studies on the subject, there was no knowledge identified by the participants on the mental calculus undertaken and the mathematical knowledge underlying it.

These data reveal this gap in the teachers' formative knowledge about the characteristics of the NDS and the properties of operations that allow us to make mental calculation strategies and their variety more

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<sup>3</sup> We are a training nominate **in** teaching to formative processes, institutionalized or not, of which these professionals, after training, are used during their performance. Training **Of** teaching is related to professional training, or rather to the preparation of this professional for the exercise of a particular function.

flexible, depending on the numbers that are involved in the operations, as evidenced in the studies presented here, evidencing the need for teaching training on the subject investigated.

## 5 CONCLUSIONS

It was considered that for the development of mental calculation skills and their relationship with the objects of knowledge, as proposed in the BNCC (BRASIL, 2017) for the initial years of elementary school, the teacher must have mastery, not only of the mathematical content to be worked but also of knowledge pedagogical to take an appropriate methodological treatment, thus promoting the development of the skills provided.

This gap in training, observed in this section of the research undertaken, about the aspects that involve the resolution of an operation, such as the characteristics of ndS and the properties of operations, leads to reflection on the way that teaching has been conducted and what learning has promoted.

In **this sense**, the formation of teaching was not sufficient to provide the knowledge necessary for the exercise of teaching to promote the development of mental calculus, which validates the importance of teaching training and implementing actions that can contribute to the teacher expanding his knowledge about this teaching content and methodological possibilities.

This cutout of the researched data brings a contribution to the process of continuous training of teachers and teachers who teach mathematics, to promote the necessary conditions for learning through the mental calculation of students.

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