SEVEN
publicações acadêmicas

Geometric transformations: Actions and applications in the final grades of elementary school

Crossref doi
https://doi.org/10.56238/ptoketheeducati-026

Jones Ueder Casarin
Mestrando em Matemática; Universidade Federal de Santa Maria/UFSM, Santa Maria, Rio Grande do Sul, Brasil.
E-mail: jonesueder@gmail.com

Valéria de Fátima Maciel Cardoso Brum
Doutora; Universidade Federal de Santa Maria/UFSM, Santa Maria, Rio Grande do Sul, Brasil;
E-mail: valeriacardosobrum@gmail.com

Karine Faverzani Magnago
Doutora; Universidade Federal de Santa Maria/UFSM, Santa Maria, Rio Grande do Sul, Brasil;
E-mail: karine@ufsm.br

ABSTRACT

The present study deals with the mathematical content of geometric transformations, bringing as a problem the fact that it is little explored, often due to the gaps in the knowledge of the teacher in relation to this theme. Given this, the main objective is to seek in historical facts the importance of this content, as well as to observe in the Brazilian curricular guidelines its prominence and its paths to be taken in the teaching of this curricular component. In addition to bibliographic research, activities with digital technologies will also be applied to work on the main isometries, such as rotation, translation and reflection, in order to contemplate the skills required by the current National Common Curricular Base (BNCC).

Keywords: Rotation, reflection, translation, transformations.

1 INTRODUCTION

Fulfilling has a very broad meaning, thus being a synonym of performing, satisfying, accomplishing, completing, among others. For a teacher within the school planning, fulfilling means much more than applying all the established contents, it is necessary that the student learns, understands, establishes relationships and knows how to apply in his daily life. Reconciling these tasks brings a conflict in the time in which each topic is covered. It is common to observe mathematics teachers from the same school who teach in different classes, but of the same grade, addressing topics at different times.

Many teachers leave this content of geometric transformations to work at the end of the school year, due to the lack of knowledge on the subject. In this scenario, the study of geometric transformations has been little explored in the classroom, as highlighted that "we found that the contents of Geometry that were not learned by teachers, were also not taught, giving rise to a vicious circle that ends up affecting generations of students who do not learn geometry" (MABUCHI, 2000, p.1).

Given this, the interest in studying the subject occurs to find its relevant importance, based on the objectives outlined by national and state curricular guidelines, such as the National Curricular

Parameters (PCNs), the National Common Curricular Base (BNCC) and also the Gaucho Curricular Reference.

The general objective of the study is to investigate the importance of the content of geometric transformations within the development of students' geometric thinking, in order to identify practices that best meet the theoretical foundations related to the subject.

As specific objectives:

- Identify the purposes presented by the national curriculum guidelines within the theme of geometric transformations;
- Use the Geogebra software in the execution of practical activities to improve the understanding of geometric transformations such as rotation, translation and reflection;
- Apply practical activities with Geogebra and manipulable didactic materials in the content of geometric transformations;

2 BRIEF HISTORY OF GEOMETRIC TRANSFORMATIONS

Many are the facts that can be highlighted when it comes to geometric transformations, because it is a millennial and global subject. Where it was studied or translated into the arts it left a meaning for humanity. It is an important connective between mathematics, art and engineering. Understanding the process of its evolution is a starting point to understand how significant this subject is within schools.

2.1 THE HISTORY OF GEOMETRIC TRANSFORMATIONS IN CIVILIZATION

The chronology of geometric transformations is very old and was revealed through several discoveries until it reaches today in schools as important school content. One of the oldest geometric demonstrations involving the symmetry of geometric transformations is a cave painting found in Bolivia. It also has the record of a piece of pottery decorated with geometric transformations that is recorded as built in the Neolithic period. In Siberia, in an excavation in a royal burial room was found the carpet of Pazynyk, dating from the fifth century BC. In Brazil, an art of indigenous tribes from the pre-colonial period, the Marajoara ceramics, was found in Pará, a work decorated by symmetries and reflections.

More recently three characters have excelled in the study and art of the geometry of transformations. Felix Klein (1849-1925), Evgraf Fedorov (1853-1919) and Maurits Cornelis Escher (1898-1972). The latter featured prominently in his works of art formed by geometric patterns such as reflection and symmetry, among other geometric operations. The Brazilian Luiz Sacilotto (1924-2003) was a painter, sculptor and draughtsman who had in his works symmetry as a striking point, also exploring optical phenomena.

2.2 THE HISTORICAL MOVEMENT OF GEOMETRIC TRANSFORMATIONS IN THE CURRICULAR GUIDELINES OF ELEMENTARY SCHOOL – FINAL GRADES

The school curriculum is structured according to debates, influenced by transformations of society. Since the 1920s there have been movements of curricular reorientation. Between the years of 1960 and 1970 the so-called Modern Mathematics influenced the teaching of mathematics, bringing school contents closer to those seen by scholars and researchers. In 1980, a document called "Agenda for Action" influenced reforms around the world. Thus, the reformulations that occurred between 1980 and 1995 had points in common, such as emphasis on problem solving, introduction of elements of statistics and probability and statistics.

In Brazil, in 1995, the National Curricular Parameters (PCNs) were initiated, a document that aimed to organize the contents. Following this process, in 1996 the Law of Guidelines and Bases of National Education (LDB) was created, a legislation that regulates the Brazilian educational system. And in 1998 the PCNs were launched, bringing an important change in the teaching of geometric transformations, geometry should not be restricted to the study of forms, but engage in the notion of position, where these forms are located and their movement within the Cartesian plane.

In 2014, a process of updating the National Curriculum Parameters began. From then on, it had several studies, debates and consultations, until in 2017 it was approved and guided the implementation of the National Common Curricular Base (BNCC). The document comes with changes in relation to the PCN, where now along with an object of knowledge, there are several skills that must be developed in students, in all the final years of elementary school.

Following the BNCC, came the creation of the Gaucho Curricular Reference, a guiding document of curricula in the State of Rio Grande do Sul. It adds new skills to the BNCC and adds regional themes such as history, culture and ethnic-racial diversity. In mathematics, the document presents the pedagogical principles, and in the part of geometric transformations, it repeats the ideas shown in the BNCC, expressing in other words the importance of this content and basing with the same intentions of the application of this content.

The main curricular guidelines of Brazil present the geometric transformations as essential, showing that it should be privileged among the contents and even when not applied, it should be treated with great concern, because it is a prerequisite in the understanding of geometric concepts. The geometric transformations are subjects present in the National Curricular Parameters, in the National Common Curricular Base and also in the Gaucho Curricular Reference, and the Homotetias are placed in the 6th year, the rotations, reflections and translations in the 7th and 8th years and in the 9th year we will have these contents as fundamental for the understanding of congruence and similarity.

3 PRACTICAL ACTIVITIES INVOLVING GEOMETRIC TRANSFORMATIONS

In the guiding documents of the curricula to be worked, the Geometric Transformations are always present. Starting from the implementation of the PCNs, in 1988, we have at least twenty years of well-defined and organized curricula, but when repairing activities related to geometric transformations, there is a scarcity compared to other mathematical contents, being another demonstration of helplessness of this content in the teaching of mathematics. Thus, in the sequence, there are some suggestions of activities and results of applications, following the skills proposed by the National Common Curricular Base

3.1 GEOMETRIC TRANSFORMATIONS IN GEOGEBRA SOFTWARE

In the National Common Curricular Base, the content of geometric transformations for the 6th year, is proposed in two skills to be developed, the first refers to "build similar flat figures, in situations of expansion and reduction, with the use of checkered meshes, Cartesian plane or digital technologies" (BRASIL, 2018, p.303) and the second does not deal directly with geometric transformations, but one can use transformations such as translation and rotation for the representation of parallel and perpendicular lines, it is addressed in "using instruments, such as rulers and squares, or software for the representation of parallel and perpendicular lines and construction of quadrilaterals, among others." (BRAZIL, 2018, p.303).

In both skills, digital technologies are approached as auxiliary instruments in the development of the proposal. Thus, through the software Geogebra was developed at the Maria Antônia Uggeri Pizetta School, in the city of Entre-Ijuís, Rio Grande do Sul, an application of the content in a class of the 6th grade of Elementary School. The activity was based on a tool of Geogebra, the homothetia. After learning about the software, students were provoked to solve tasks on their phones or computers. With each step taken, the teacher projected the result on the board and there was the conference and understanding of what was being done or what was in disagreement.

Activity: Homothety in Geogebra:

1) Select the "Point" tool and mark five points: A (2,3), B (4,3), C (3,5), D (0,3) and E (0,6).

2) Select the "polygon" tool and click on points A, B, and C.

3) To make the homothety we select the tool "homotetia", then the point D and the polygon formed, after that put the number 2 in the tab that will appear.

4) Select the point E and the polygon formed, after that put the number 0.5 in the tab that will appear.

5) Which polygon will appear?

6) What happens when the number 2 is placed on the tab that appears after the polygon and point D are selected?

7) Explain what happens when the 0.5 is placed on the tab that appears after selecting the polygon and the E point?

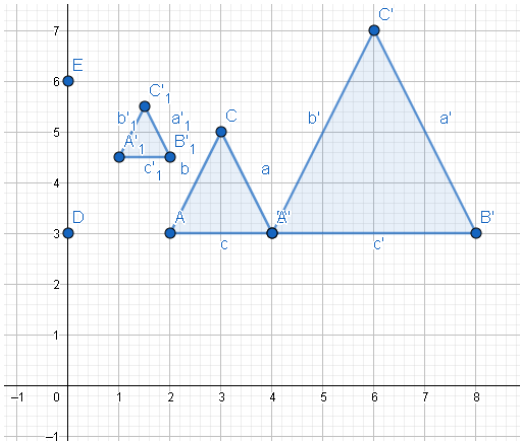
8) For which values placed in the tab will we have a polygon magnification?

9) For which values placed in the tab will we have a reduction of the polygon?

10) Which value placed on the tab will keep the same polygon, in the same place and at the same size?

At the end of the activity, students should arrive at the result of the figure below:

Figure 1- Homothetia in Geogebra



Source: Prepared by the author

The realization of the activity brought an intense movement of the students, unlike what usually happens in other mathematics classes.

As not everyone had access devices to Geogebra, the creation of groups helped them in carrying out the tasks.

They performed the activities with a lot of autonomy, and when they arrived at items 8, 9 and 10, they began to conjecture, simulating values.

Every time they simulated the values opened a tab with the word "factor", thus questioning the teacher about what this word meant.

The teacher then explained that it would be the result of dividing the corresponding sides of the polygons and that in the 9th grade they would understand it as a constant of proportionality in the similar polygons.

Figure 2- Students comparing the results on the smartphone with the projection

Source: Image of the author

In the 7th year three skills proposed by BNCC involve the content of geometric transformations, the first is about "performing transformations of polygons represented in the Cartesian plane, resulting from the multiplication of the coordinates of their vertices by an integer" (BRASIL, 2018, p.309), while the second suggests "recognizing and presenting, in the Cartesian plane, the symmetrical of figures in relation to the axes and the origin" (BRASIL, 2018, p.309), the last one discusses "recognizing and constructing figures obtained by translation, rotation and reflection symmetries, using drawing instruments or dynamic geometry software and linking this study to flat representations of artwork, architectural elements, among others" (BRASIL, 2018, p.309).

It should be noted that it is also suggested the use of software for the development of one of the skills. Thus, an activity involving Geogebra and geometric transformations was also applied in the 7th grade of the same school in the municipality of Entre-Ijuís. The practice consisted of the same dynamics applied to the 6th grade, but aiming to develop the skill that suggests the use of dynamic geometry software.

Activity: Reflection on Geogebra:

1) Select the "Point" tool and mark three points: A (1,3), B (2,1) and C (4,2)
2) Select the "Polygon" tool and click on points A, B and C
3) To make the reflection select the tool "Reflection in relation to a line", click on the polygon formed and the axis of the ordinates
4) Click on the tool "Reflection in relation to a line", click on the polygon and the axis of the abscissas
5) Select the "Point" tool and mark the D point at the source (0,0)
6) Select the tool "Reflection in relation to a point", click on the polygon formed and on the point D
7) What polygon was formed between points A, B and C?

8) The polygon generated in the reflection of the ABC polygon and the axis of the ordinates is in which quadrant?

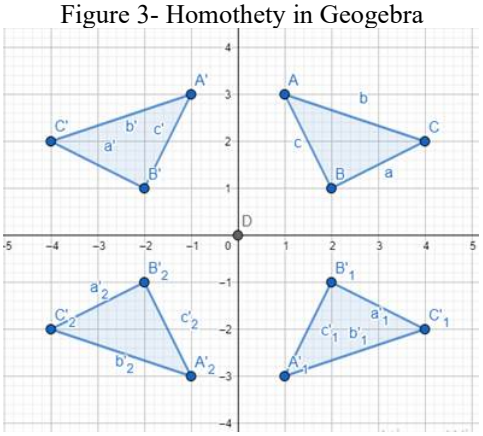
9) The polygon generated in the reflection of the ABC polygon and the axis of the abscissas is in which quadrant?

10) The polygon generated in the reflection of the ABC polygon and the point D, is in which quadrant?

11) What do you understand about reflection?

At the end of the activity students should arrive at the result below:

Figure 3- Homothety in Geogebra



Source: Prepared by the author

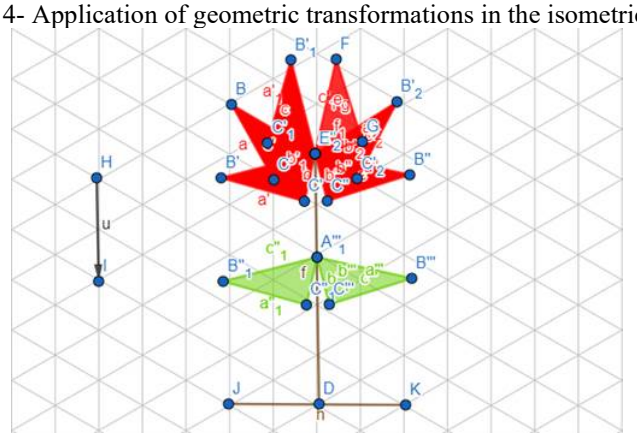
The results of the presentations were also surprising, with a lot of student involvement, since the 7th grade class is composed of several students with grade/age distortion and with two inclusion students. The proposal even brought the participation of the two students with inclusion, even though they were at the literacy level, with the help of colleagues they performed the Geogebra tasks. At first, the class had a hard time, because during the 6th grade their studies were affected, with hybrid teaching, due to the COVID 19 pandemic. Once they understood the process, everything became easier.

Similarly, to the reflection activity, the constructions of rotation and translation in Geogebra were also performed. The latter, with very fast realization. Still, it is easily observable for those who conduct daily mathematical activities, that the use of digital technologies provides a greater acceptance of teaching and consequently, a better learning of the contents. It should be noted that the contents of geometric transformations were being worked on in an unprecedented way, because the study plan of the municipality began to have well defined this part of mathematics after the launch of the BNCC.

In the eighth year also the BNCC proposes a skill on geometric transformations, and again suggests the use of software. The ability is described as "recognizing and constructing figures obtained by compositions of geometric transformations (translation, reflection and rotation), with the use of drawing instruments or dynamic geometry software" (BRASIL, 2018, p.315). At this stage a suggestion of activity, with the use of Geogebra, would be the creation of a figure or a logo, but using

the isometric mesh and not the checkerboard mesh. This would bring a new way of working with Geogebra and introduce a mesh that will be part of the content of the 9th grade, in the construction of perspective objects, to recognize orthogonal views of spatial figures.

Figure 4- Application of geometric transformations in the isometric mesh



Source: Prepared by the author

In the 9th year, the geometric transformations are not mentioned directly, but at this stage it will be necessary this content to understand the similarity and congruence of figures and also in the demonstration of metric relations of the right triangle. It would also be important to use Geogebra to understand the similarity and congruence of polygons, being able to observe the measurements of the sides and angles, their congruences, proportionality coefficient among others.

4 FINAL CONSIDERATIONS

The inclusion of geometric transformations in the study of geometric concepts has been greatly emphasized in curricular guidelines in Brazilian education. Despite the relevant importance, this content is little explored by the teacher, often due to the lack of knowledge on this subject, bringing with it a great loss in the student's learning regarding the understanding of geometric concepts, such as: Cartesian plane, angles, congruences, similarity of figures, metric relations, among others.

It is necessary to note that all mathematical content is important in its essence and that one content complements the other. This fact can be verified within the applications, suggestions and also within the order of skills treated by BNCC, where the pedagogical strategy known as spiral education, with the revisiting of the contents, is proposed.

It is widely known that the study of geometry carried out by the initial formation of the teacher, is not enough to solve the difficulties they will face when teaching the contents of geometry. Thus, it is believed that the study of geometric transformations in a continuing education of the teacher, would make him able to apply the content as proposed by the BNCC.

In order to help elementary school teachers to work the content of geometric transformations in a more dynamic way, as proposed by BNCC, activities were developed in this work with students

of the 6th and 7th grades of elementary school, addressing topics such as homothecy and isometries using the Geogebra software.

REFERENCES

BRASIL. Ministério da Educação. Base Nacional Comum Curricular. Brasília, 2018.

BRASIL. Parâmetros Curriculares Nacionais: terceiro e quarto ciclos: matemática. Brasília: MECSEF, 1998

MABUCHI, S.T., Transformações geométricas: de um conteúdo ainda não incorporado às práticas escolares nem à formação de professores. 2000. 1 f. Dissertação (Mestrado em Educação Matemática) – PUC, São Paulo, 2000.

RIO GRANDE DO SUL. Referencial Curricular Gaúcho. Secretaria de Estado da Educação: Porto Alegre, 2018

TRANSFORMAÇÕES geométricas nos programas de matemática do ensino básico e secundário. Departamento de matemática, Universidade de Coimbra. Disponível em: < http://www.mat.uc.pt/~mat0829/Transformacoesgeometricas-2.pdf>. Acesso em: 15 dez. 2022