RECREATIONAL MATHEMATICS IN THE TRAINING OF MATHEMATICS TEACHERS FROM THE PERSPECTIVE OF THE THEORY OF OBJECTIFICATION

https://doi.org/10.56238/sevened2024.031-051

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ABSTRACT

Recreational Mathematics is a methodological approach in Mathematics Education and involves games, puzzles, Recreational Problems, topological curiosities, magic, art, origami, in addition to other tasks of a playful-pedagogical nature. Thus, our research aimed to investigate the contributions of Recreational Mathematics to the training of Mathematics teachers, using the Objectification Theory as a theoretical and methodological basis. To achieve our objective, the methodology used was qualitative and exploratory. In this way, we are based on the theoretical-methodological references of authors who deal with Recreational Mathematics and the Theory of Objectification, such as Bártlová (2016), Nunes (2019), Bezerra (2021; 2022; 2023) and Radford (2018; 2020; 2021). The results demonstrated the interrelationship between the Objectification Theory and Recreational Mathematics, evidencing its implications in teacher training and future pedagogical practice.

Keywords: Recreational Mathematics. Theory of Objectification. Mathematics Teacher Training.

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INTRODUCTION

Teacher training is a complex process, however, crucial for the development and professional performance of the Mathematics teacher. In addition, both the initial and continuing training of teachers are marked by problems such as the distance between research, teaching and extension, in which it strengthens educational practices that dissociate theory from practice (Albuquerque; Gontijo, 2013). For the authors, the non-incorporation of research results in the area of Mathematics Education disfavors teacher training.

We defend an emancipatory education, where teachers are understood as unfinished human beings, in movement and in constant evolution; for this, we chose the Theory of Objectification as a theoretical-methodological contribution. Why the Theory of Objectification? Because it is a theory of teaching and learning belonging to the field of contemporary sociocultural theories, which considers learning as a simultaneous process, in which teachers and students work collectively, for the mobilization and encounter with knowledge (objectification process) and the transformation and co-production of subjects (subjectivation process), idealized by professor and researcher Luis Radford.

The Theory of Objectification can offer Mathematics teachers in initial and/or continuing education opportunities to think, dialogue and produce reflections on teacher training based on the fundamental principles of joint work and Community Ethics. From this perspective, the Theory of Objectification offers modes of human collaboration, responsibility, commitment and care for the other.

Thus, a methodological approach in Mathematics Education that we chose to work on in the training of Mathematics teachers, based on the Theory of Objectification, was Recreational Mathematics that can contribute to more general purposes, for example: to promote learning; relate contents studied in the classroom with the History of Mathematics; to provide entertainment and enthusiasm, inside and outside the classroom, and to serve as a means of popularizing Mathematics. However, we want to highlight some potentialities that Recreational Mathematics can promote in pedagogical practice: pleasure, joy, fun, and other positive dimensions in the classroom (Bezerra, 2021).

In this context, Recreational Mathematics provides a variety of tasks of a recreational, pedagogical and historical nature that can be used at different levels of education. In this way, it presents itself as an important methodological approach to show students that Mathematics can be fun and pleasurable. Therefore, the use of this tool in classes can promote changes, both in the structure of the classroom, as well as in the way

of teaching and learning mathematical content, showing the playful and creative side of Mathematics.

As for the insertion of Recreational Mathematics in the teaching of Mathematics, it is justified, not only aiming at the application and/or exploration of specific contents of Mathematics, but aiming at the development of skills such as: concentration, persistence, intuition, creativity and dialogue that will help prepare students, not only for more advanced levels of schooling, but to act as critical citizens, reflective, conscious, and participative in the face of the challenges that life proposes to them (Bezerra, 2021; 2022).

In this circumstance, we intend to present and propose the Theory of Objectification and Recreational Mathematics in the training process of Mathematics teachers, based on the principles that involve joint work and Community Ethics.

Thus, considering the arguments exposed, the guiding question of our investigation is: *what contributions of Recreational Mathematics to the training of Mathematics teachers, having as theoretical-methodological support the Theory of Objectification?* In this context, to answer this question, the central objective of the investigation was to *investigate the contributions of Recreational Mathematics to the training of Mathematics teachers, using the Objectification Theory as a theoretical and methodological basis.*

As can be seen, the set of ideas and choices outlined so far is based on the articulation between the Theory of Objectification and Recreational Mathematics in the training of Mathematics teachers. Next, we will present a deepening of these two theoretical axes.

OBJECTIFICATION THEORY AND RECREATIONAL MATHEMATICS

Our theoretical-methodological options are based on the Theory of Objectification and Recreational Mathematics from researchers such as Bártlová (2016), Radford (2018; 2020; 2021), Bezerra (2021; 2022; 2023), Paiva and Bezerra (2024), Bezerra and Silva (2024), among others; that seek in their studies and research, to promote a reflection on the practice of the Mathematics teacher regarding their professional skills, their social role as an educator and the ability to insert themselves in the various realities with sensitivity to interpret the actions of the students.

The Theory of Objectification (OT) is a contemporary sociocultural educational theory, inspired by the philosophical works of Hegel (1770 – 1831), Marx (1818 – 1883), contemporary philosophers such as Ilyenkov (1924 – 1979) and the historical-cultural school of Vygotsky (1896 – 1934).

> The Theory of Objectification (OT) is situated in a different educational project: it sees the objective of Mathematics Education as a political, social, historical and cultural effort that aims at the dialectical creation of reflective and ethical subjects who take a critical position in historically and culturally constituted mathematical discourses and practices, and who ponder new possibilities of action and thought (Radford, 2021, p. 38).

The Theory of Objectification brings contributions to the organization of teaching practice in Mathematics Education and is applicable, in general, in Education. The Objectification Theory considers learning as a simultaneous process, in which teachers and students work collectively – Joint Labor (Radford, 2020).

Learning Mathematics involves emotions and affections in ways that affect us as human beings. For this reason, in the classroom, we not only produce knowledge, but also subjectivities. The processes of subjectivation, in turn, are those of the incessant creation of the subject, the continuous creation of a historically and culturally singular, unique subject (Radford, 2018).

The Theory of Objectification elaborated a concept of activity in the classroom coherent with a historical-cultural view of the human being. Activity is, therefore, the process by which knowledge is materialized in knowledge. This activity actualizes knowledge and brings it to life (Radford 2020). Thus, what makes learning possible is human and practical activity. In this way, the processes of objectification and subjectivation occur simultaneously in a continuous and intertwined way during the activity considered social, historical and cultural.

The classroom activity, according to Radford (2020), is called joint labor. Thus, joint labor is the main ontological and epistemological category of the Theory of Objectification, considering the classroom activity as its unit of analysis. However, the role of language, signs, artifacts and the body is not discarded in the processes of transformation of knowledge into knowledge. In this theory, language, signs, artifacts and the body are understood not as mediators, but as part of the activity of individuals.

Joint Labor, according to Bezerra (2021), is the work by which teachers and students assert themselves in their production and act as human beings in what they do, therefore, joint Labor is the type of activity that is not alienating, as its characteristic is the practice of a Community Ethic and it is this ethics that guides the didactic action in Mathematics classes.

From the perspective of the Theory of Objectification, there is the possibility of thinking about the teaching and learning of Mathematics, incorporating the ethical dimension with the understanding of didactic relations and the constitution of subjectivities. Community Ethics is based on three vectors: responsibility, commitment, and care for others (Radford, 2021). Community Ethics can allow forms of collaboration that are granted

to the extent that there is a commitment to work together, to care for others and to assume responsibility towards others.

Recreational Mathematics is a methodological approach in Mathematics Education that we have chosen to work on in the initial and/or continuing training of Mathematics teachers. Recreational Mathematics, in turn, is a strand that can associate Problem Solving, the use of Games, the History of Mathematics, Investigations and Argumentation in Mathematics, Digital Technologies, among others.

Recreational Mathematics can be an important methodological approach to the teaching of Mathematics, because it can be seen as a playful way of presenting Recreational Problems, games, puzzles, in addition to other tasks of a playful-pedagogical nature and not only for fun, but also to arouse curiosity and the challenge to develop logical-mathematical reasoning (Bezerra, 2022). In this sense, its use can contribute to making Mathematics classes more dynamic and collaborative for students, and helping them to perceive Mathematics as a science, whose practice can be pleasurable.

Costa (2014), presents us with the following definition of Recreational Mathematics: "[...] recreational mathematics is that mathematics that challenges us to think, entertains us and amuses us when we think about it" (2014, p. 6). As Nunes (2019, p. 23) argues, "Recreational Mathematics (MR) is a part of the usual mathematics, "serious" mathematics, which carries a trace of fun". In this way, Recreational Mathematics can serve as a bridge for the elaboration of mathematical concepts and its scope goes beyond games, puzzles, problems, challenges and competitions, as Recreational Mathematics is very broad in the field of Mathematics.

Recreational Mathematics is on the border between the four aspects: the scientific-popular, the fun (entertainment), the pedagogical and the historical – which are interconnected and influence each other. The four aspects overlap considerably, so that there are no clear boundaries between them and serious mathematics (Bezerra, 2021).

The scientific-popular aspect makes Recreational Mathematics fun and popular, that is, the corresponding problems must be understandable to an interested layman, although the solutions may be more difficult. In the context of the second aspect, fun (entertainment), Recreational Mathematics is understood as a Mathematics that is used as a detour to serious Mathematics, that is, for fun.

In the third aspect, the pedagogical one, Recreational Mathematics can be used for teaching purposes. Recreational Mathematics can provide a variety of problems for investigations in the training of Mathematics teachers and in the classroom in Basic Education. The historical aspect refers to the fact that Recreational Mathematics is old and

there are indications that this tool originated thousands of years ago (Bartlová, 2016). Another meaning that we can attribute to the historical aspect is the possibility of being able to use Recreational Mathematics tasks to study the History of Mathematics.

It is important to highlight that the Objectification Theory helps to materialize Recreational Mathematics in the classroom, for example, Recreational Mathematics tasks are, in general, open, because they help to promote a deepening of mathematical thinking in relation to a certain content; it stimulates collective discussion and motivates: student participation in the process; in the interaction with knowledge, with other colleagues and the teacher, and in reflections on the actions carried out in the problem-solving process. In addition, Recreational Mathematics can contain elements of a historical and social nature that promote the discussion of ethical and reflective positions, among others.

To guide the elaboration of the main tasks of Recreational Mathematics: mathematical games, mathematical puzzles and Recreational Problems, the Theory of Objectification was in charge. Thus, to present Recreational Mathematics to the students of the Mathematics Degree Course, at the Federal Institute of Rio Grande do Norte (IFRN), Ceará-Mirim Campus, aiming at the knowledge of this methodological approach and its application in the classroom.

Therefore, Recreational Mathematics tasks should provide didactic possibilities beyond the playful, recreational, and motivational aspects, focusing on mathematical investigation, problem-solving strategies, in addition to inducing the search for strategies and exploring mathematical concepts involved in the tasks (Bezerra, 2022; 2023). The teacher must feel prepared to elaborate the Recreational Mathematics tasks and explore them in the classroom.

In the next section, we present the methodology of this research.

METHODOLOGY

The work developed in this study is characterized by a methodological-qualitative approach, of an exploratory nature, because according to Gil (2008, p. 41), this type of research "[...] aim to provide greater familiarity with the problem, with a view to making it more explicit or to constitute hypotheses".

Therefore, it is a qualitative research that presents a methodological alternative by proposing the Theory of Objectification as a theoretical-methodological framework. The Theory of Objectification presents its own methodology for the analysis of the teaching and learning process and in the understanding of how students become aware of scientific

knowledge and position themselves critically and ethically during and after interactions in the classroom.

The research was developed in 2023, with the undergraduates of the Mathematics Teaching Degree Course, enrolled in the disciplines of Mathematics Teaching Methodology I and II, at the Federal Institute of Rio Grande do Norte (IFRN), Ceará-Mirim Campus. Thus, we developed our research, according to the following steps.

- Theoretical study on Recreational Mathematics, for this, we carried out a bibliographic survey in Theses and Dissertations on the website of the National Base of the Digital Library of Theses and Dissertations (BDTD).
- Selection of Recreational Mathematics tasks in Theses and Dissertations related to Recreational Mathematics, to constitute a corpus to present to Mathematics teachers, in initial training.
- Analysis, creation, construction and application of Recreational Mathematics tasks from the perspective of the Objectification Theory.
- Analysis of the results produced during the research in the light of the fundamental principles of joint labor and the Community Ethics of the Theory of Objectification.

Therefore, we will present the results and discussions of this research.

RESULTS AND DISCUSSIONS

In this section, we present the results of some actions that were developed in the disciplines of Mathematics Teaching Methodology I and II, using Recreational Mathematics tasks based on the Theory of Objectification, in the Mathematics Teaching Degree Course, IFRN, Ceará-Mirim Campus, in the initial training of Mathematics teachers.

In 2023, we carried out a bibliographic survey of Theses and Dissertations to learn more about research related to Recreational Mathematics. Subsequently, the most relevant studies to our study were analyzed (Costa, 2014; Bartlová, 2016; Nunes, 2019; Bezerra, 2021). Thus, we obtained the conceptions and aspects of Recreational Mathematics, the main tasks of Recreational Mathematics (games, mathematical puzzles and Recreational Problems), as well as works and authors that contributed to the dissemination of Recreational Mathematics. Thus, the theoretical bias on Recreational Mathematics is one of the contributions of our research.

Following the readings and reflections related to Recreational Mathematics, it was possible to observe that some of the studies already bring experiences of tasks applied in the classroom at different levels of education (Bezerra, 2021). In this way, we have the

material of Recreational Mathematics to present to Mathematics teachers, in initial training, having as a theoretical-methodological contribution the Theory of Objectification.

Thus, in the disciplines mentioned above, we work with tasks involving the Game Cover Twelve, Tangram, Tower of Hanoi, Origami, Recreational Problems extracted from the work *Problems to stimulate young people* by Alcuin of York (735 – 804), the work *De Viribus Quantitatis* by Luca Pacioli (1445 – 1517), the work *Liber Abaci* by Leonardo Pisano (1170 – 1250) and the work *The Man Who Calculated* de Mello e Sousa – Malba Tahan (1895 – 1974), so that teachers in initial training understand mathematical content and the potential of Recreational Mathematics. In this perspective, Bártlová (2016) states that games, puzzles and Recreational Problems satisfy the need for fun, joy and pleasure, in addition to the desire to achieve mastery over challenging subjects, or simply test our intellectual capacities.

In this way, by incorporating Recreational Mathematics into their methodologies, teachers can develop more dynamic and collaborative didactic approaches. This fosters a learning environment where students feel more motivated to participate and explore mathematical concepts.

With regard to the development of Recreational Mathematics tasks, we were able to observe potentialities that Recreational Mathematics promotes: joy, pleasure, motivation, entertainment, enthusiasm and fun. A diversion that can provide an experience of cooperation between Mathematics Degree students, thus facilitating the awareness of the mathematical elements mobilized in the joint Labor (Bezerra, 2021). In this way, the activity in the classroom is seen as a joint work of teacher-students and student-students and supported by a Community Ethic.

We observed that the Mathematics undergraduates, during the development of the Recreational Mathematics tasks, worked through the shoulder to shoulder activity (Joint Labor), with commitment, dialogue, responsibility and care for the other. Commitment to the fulfillment of the task, respect, care for others and responsibility (response) towards the other, which Radford (2021) calls Community Ethics.

The Theory of Objectification values the joint work between teachers and students, favoring knowledge and the transformation of the subject. Therefore, we reaffirm the importance of the Objectification Theory for our investigation, mainly as an object of pedagogical reflections so that Mathematics teachers can get to know Recreational Mathematics and position themselves on how and why this methodological approach should be present in Mathematics classes.

In view of the above, we highlight some contributions of Recreational Mathematics to the training of Mathematics teachers from the perspective of the Objectification Theory: to offer teachers a notion of what the Objectification Theory and Recreational Mathematics are; the Objectification Theory contributes to the training of more critical and reflective teachers, in the improvement of the quality of Mathematics teaching and learning, in addition to the expansion of the use of pedagogical methods in public schools; work on recreational, historical and pedagogical aspects of Recreational Mathematics; development of mathematical topics; logical reasoning; playful, recreational and pedagogical character and; possibilities and proposals for the insertion of Recreational Mathematics tasks in the training of Mathematics teachers.

As a result, we expect that Mathematics teachers acquire skills and abilities to use Recreational Mathematics tasks in a critical and responsible way. In addition, understand the potentialities, advantages and disadvantages of implementing Recreational Mathematics in the Mathematics classroom, with the objective of improving the quality of teaching and learning.

CONCLUSION

In this work, we seek to broaden the view on the following question: *what contributions of Recreational Mathematics to the training of Mathematics teachers, having the Theory of Objectification as theoretical-methodological* support?To answer it, we outlined as our main objective: *to investigate the contributions of Recreational Mathematics to the training of Mathematics teachers, using the Theory of Objectification as a theoretical and methodological basis.*

The Theory of Objectification provides a new perspective for teacher education, understanding it as a dynamic, collective and creative process. This approach has the potential to produce modes of human collaboration and interaction that promote: critical posture, ethics, solidarity, responsibility, and care for the other. In this sense, the Theory of Objectification is configured as a proposal for improvement in the teaching and learning of Mathematics based on the forms of human collaboration – on joint work and on Community Ethics.

Recreational Mathematics encourages teachers to explore problems in an unconventional way, promoting critical thinking and problem-solving skills. This helps in the training of Mathematics teachers who can teach students to approach mathematical situations in a creative way.

Another outcome is to contribute to the training of critical, reflective and ethical teachers and students. Thus, we hope that this work is useful, both for students of the Mathematics Teaching Degree Course, as well as for teachers, researchers, but also for other people interested in teacher training. Therefore, in the field of teacher training there is still a lot of potential to be explored about Recreational Mathematics in the light of the Theory of Objectification.

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