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CHAPTER 8

BASIC EPIDEMIOLOGICAL MODELS IN A PROPOSAL FOR THE TEMPORAL EVOLUTION OF AIDS

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ABSTRACT

From the 80's to the present day, there has been great progress in research regarding the diagnosis, treatment and control of AIDS, but this disease still affects the Brazilian population with intensity. The HIV virus acts on the immune system, leading to the emergence of opportunistic diseases. Antiretroviral therapies allowed patients to maintain a healthy life for a long time. However, some recent research has shown that this virus may be responsible for premature aging of infected people, causing the degenerative character of the disease to be widely discussed. Just as health scholars have been researching how to contain this syndrome more effectively, mathematicians have developed some epidemiological models, formed by systems of ordinary nonlinear differential equations. Therefore, mathematical modeling has been an important tool in Epidemiology. The basis of our studies on the evolution of AIDS begins with a system, representative of the IS model, built for the city of Manaus, capital of Amazonas, between 2009 and 2014. For this, we carried out a bibliographic survey on the basic models SI, SIS, SIR, SIRS and SIRV, analyzed the criteria for the stability of the critical points, in addition to observing the behavior of the graphs generated by the temporal evolutions, built in the Octave software. With this, we are able to make some predictions about the number of infected, susceptible, or even how many would be recovered or vaccinated, in this process of evolution. We will start from the simplest model, where there is no form of recovery, until the conclusion of the studies with the post-vaccination model.

Keywords: HIV modeling. Temporal Evolution of AIDS. Modelling of Evolution to AIDS. HIV evolution. Basic Models for the evolution of AIDS.

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INTRODUCTION

Several researchers are using mathematical models to analyze the behavior of the HIV virus. A tool that helps this study is mathematical modeling. Modeling is a fundamental step in this investigation process, as it allows a better understanding and analysis of the situation generated by the spread of this infectious disease. The dynamics between the compartments are studied by the researchers, who have an important mission for Epidemiology, which consists of making predictions about the evolution of this and other diseases, based on graphs and numerical data. The continuous transfer between these groups of individuals organized into compartments can represent something extremely beneficial or very harmful to health.

AIDS is represented by a model called SI, where there are only susceptible and infected individuals, so there are only two compartments S and I. The temporal evolution of this model will show us that after a period, all susceptible individuals, that is, all individuals who maintain some kind of intimate relationship with the infected, become contaminated. Consider intimate relations to be unprotected sexual relations, the use of contaminated syringes, blood transfusions, non-sterile invasive instruments, from mother to child during pregnancy and breastfeeding (LOPES and PRATA, 2019).

> Although since the identification of the first case of Acquired Immunodeficiency Syndrome (AIDS) in the early 1980s, much progress has been made in terms of diagnosis, treatment and surveillance, the disease still stands out among important infectious diseases due to the magnitude and extent of damage caused to the global population. (BECK, 2014, p.13).

Mathematical modeling in Epidemiology, according to Quadros (2013), is developed through the study of equations that describe the interaction between the population of a region and the environment in which it lives, resulting in a detailed analysis of the disease. The importance of this study is due to the fact that the more we know about the disease and how it spreads, the more effective the methods will be to prevent its transmission, and even the study of preventive actions, such as vaccination campaigns.

According to the UN (2018 apud LOPES and PRATA, 2019, p. 70), in Brazil, Amazonas appears in third place in the ranking of Brazilian states with one of the highest confirmed cases of HIV. And Manaus occupies the fourth position in the list of Brazilian capitals with the highest numbers of people infected by the virus. According to the Ministry of Health (2019 apud LOPES and PRATA, 2019, p. 69), HIV viruses attack the cells that are fundamental for immunity in the human body. The virus is able to alter the DNA of CD4+ T lymphocyte cells and make copies of itself. After multiplying, it breaks the lymphocytes in search of others to

continue the infection. According to the HIV/AIDS Epidemiological Bulletin 2013, about 720 thousand people infected by the HIV virus (human immunodeficiency virus) live in Brazil today, and only 436 thousand have a link to any health service.

According to Beck (2014, p.50), "CD4+ cell count is used as an immunological parameter in order to assess disease progression and/or the effectiveness of antiretroviral therapy, in addition to being an important predictor of the risk of developing opportunistic infections and AIDS mortality".

> AIDS is the worsening of HIV infection, marked by a great impairment of the patient's immune system, leading to the emergence of opportunistic diseases caused by viruses, bacteria, protozoa, fungi and neoplasms. Due to advances in antiretroviral therapies, patients living with HIV can stay healthy for many years. However, recent studies have shown that infection by the HIV virus initiates a faster aging process compared to healthy people, so that today the degenerative character of the disease is already discussed. A factor that possibly contributes to this process is the cellular response added to the individual's age – which occurs prematurely, in response to stress – and its relationships with comorbidities and antiretroviral therapies. (CAMPANY.et.al, 2021, p. 375).

In view of the current scenario, this study related to AIDS in Manaus becomes relevant, as it proposes more simplified initial models, advancing to a more complex model, resulting from the future possibility of vaccination.

Are control measures, such as vaccination, a way to control the transmission of diseases? By reducing the number of susceptible people, immunizing them, will we have as a consequence a decrease in the incidence of the disease? (VIEIRA, 2016). We believe so, because when we analyze the temporal evolutions of the SIRV model, we realize that with vaccination, we reduce the possibility of susceptible people becoming infected, thus avoiding the growth of those infected.

To start our evolution proposal, we present the formulation of the simplest model, progressively advancing to more complex models, until we reach the SIRV model. What we are proposing in this article is a proposal to introduce a vaccine for AIDS, or another form of treatment, in addition to antiretroviral drugs that attenuate the effects of the disease, but do not allow the patient to recover, acquiring immunity, or at least to return to the initial situation of susceptibility to the disease.

SI MODEL

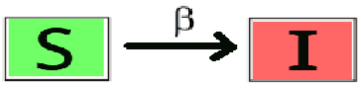
We started our evolution starting from the AIDS problem situation in Manaus between 2009 and 2014, represented by the SI model, which is the most simplified epidemic model, because in a population with N inhabitants, we have only susceptible individuals S and infectious I where S+I=N, at the time t considered. In this way, an individual infected with a

contagious disease is introduced into a population of susceptible people, and a susceptible one, once infected, becomes infectious. According to Teles (2020), epidemiological models, represented by compartments S, I are powerful mathematical tools for the dynamics of complex systems and their interactions. These systems have important applications in different scientific areas and are fundamental for decision-making in the area of public health and biomedical research.

> The rates of HIV infection throughout Brazil are a cause for great concern, especially in the structural and health spheres. We know that the northern region is one of the regions that suffer the most from the lack of public health services, so an epidemiological study in this region is of paramount importance due to the lack of population. (LOPES and PRATA, 2019, p. 72).

The contagion rate β represents a percentage proportional to these encounters between susceptible and infectious. According to Lopes and Prata (2019), there was initially 0.036% of the population of Manaus infected, while 99.964% were susceptible, in addition, the contagion rate obtained was 0.1454. According to Quadros (2013, p. 24), "in this model there is no recovered and everyone in the population is either susceptible to the disease or infected. An infectious individual, once infected, never recovers from the disease."

Figure 1: Compartment diagram representing the SI model



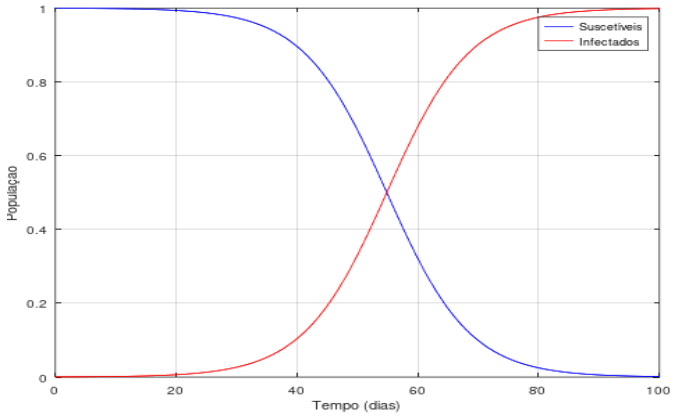
Source: Author

We can describe the equations of the differential system (Equation (1)) of the SI model, from the two compartments S and I (Figure 1). This is the most simplified system in our evolution.

$$\begin{cases} \frac{dS}{dt} = -\beta . S . I \\ \frac{dI}{dt} = \beta . S . I . \end{cases} \quad (1)$$

The algorithm used in the Octave Software, to construct the temporal evolution of AIDS in Manaus, considers this variation to calculate the iterations. After 80 days, that is, after the eightieth iteration, the population of susceptible people progressively decreases, until it becomes non-existent. As a result, the population of Manaus, with some type of contact with infected individuals, becomes infected, starting to cause infections in other susceptible individuals, as we can see in Figure (2).

Figure 2: Temporal evolution of AIDS in Manaus - SI model



Source: Author

SIS MODEL

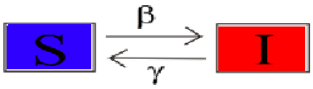
Now, imagine if in the case of AIDS, there was a possibility of recovery for infected individuals, and they became susceptible again. This, for now, is not true, it is just a supposition. We would have, in this case, a representation of the SIS model instead of the SI, whose system is represented in Equation (2). γ

Notice that the term referring to the recovery rate appeared, this causes individuals to be transferred from compartment I to compartment S.

$$\begin{cases} \frac{dS}{dt} = -\beta . S . I + \gamma . I \\ \frac{dI}{dt} = \beta . S . I - \gamma . I . \end{cases} \quad (2)$$

In the SIS model, after a period, infected individuals recover and become healthy, returning to the susceptible group, which does not happen in the SI model. According to Luiz (2012, p.31), "infected individuals, when they recover, do not acquire immunity and return to the susceptible class". Therefore, when there is the possibility of recovery, but without immunity, we have the model called SIS (Figure (3)).

Figure 3: Compartment diagram representing the SIS model

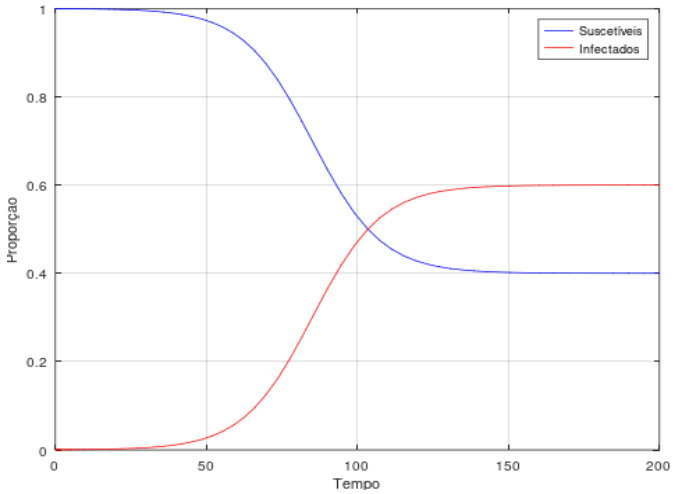


Source: Author

However, if we take the ratio between the contagion rate β = 0.1454 and γ = 0.05816, we get the basic reproduction number.

It represents an average value of cases generated by a single infected person in a susceptible population. As R0 = 2.5, there is an increase in the disease, since R0 >1.

Figure 4: Temporal evolution of AIDS Idealized in Manaus - SIS model

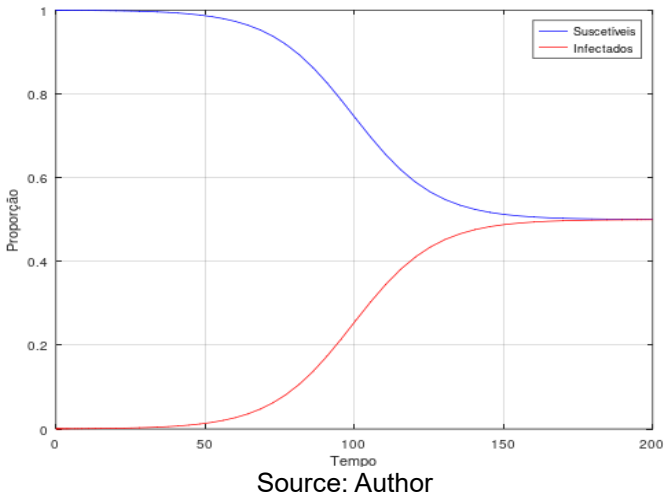


Source: Author

It is easy to see that after 100 days, that is, after the thousandth iteration, those infected start to assume values higher than those of susceptible ones. This change in the model from the IS to the SIS, would mean, in the case of the chosen recovery rate, a stabilization of the number of infected people for 60% of the population, where the remaining 40% would be susceptible, after 150 days, instead of the entire population, to become contaminated in 80 days, as in the SI model. There is a decrease in the susceptible and an increase in the infected, until the situation is stabilized. See Figure (4).

The lower the R0, this percentage of stability tends to decrease. For example, if the recovery rate γ increase and becomes equal to 0.0727, the coefficient R0 becomes equal to 2, with this, the curves of infected and susceptible individuals reach a stabilizing percentage of around 50%, as observed in Figure (5). Which means that the higher the recovery rate, the lower the percentage of infected that will remain constant in future iterations.γ

Figure 5: Temporal evolution of AIDS Idealized in Manaus - SIS model



Source: Author

SIR MODEL

In this same context, and if we started to imagine, that there is a recovery, through some type of treatment, where the recovered individual is not susceptible again. We would then have the SIR model, for an Idealized AIDS. Considering the same SIS rates β = 0.1454 and γ = 0.05816, let's take another step in our evolution. In the SIR model, according to Luiz (2012), the population is made up of susceptible people who contract the infectious disease, becoming infected, and after a period, acquire immunity, but latent periods or isolation are not considered. Note that we are not yet dealing with any form of vaccination. See Figure (6).

Figure 6: Compartment diagram representing the SIR model

Source: Author

As a result, a portion of those infected now belong to the class of those recovered. This model is different from the models previously studied, as there is the possibility of recovery, with the immunization of some individuals.

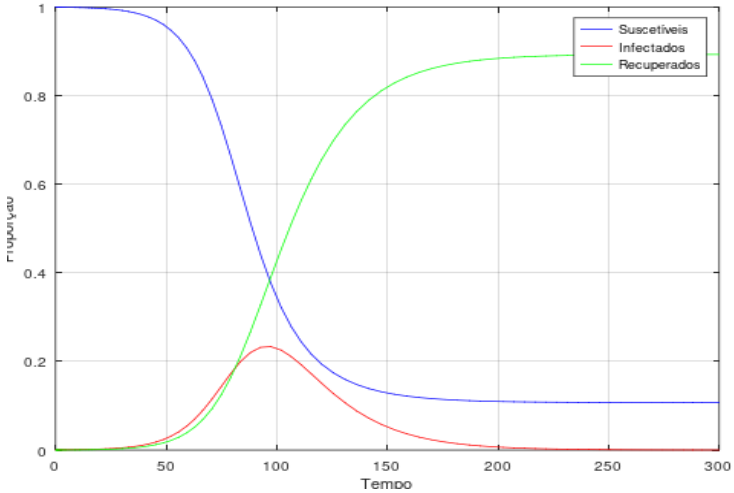
The system of differential equations (3) represents this model.

$$\begin{cases} \frac{dS}{dt} = -\beta . S . I \\ \frac{dI}{dt} = \beta . S . I - \gamma . I \\ \frac{dR}{dt} = \gamma . I . \end{cases} \quad (3)$$

What we propose is a simulation of something unreal, at least until the writing of this article, where a portion of the infected recovers and leaves this category, moving to the group

of those recovered. This can happen, due to some treatment that cures the patient, and he is not vulnerable again. This type of discovery would also be a great advance for society.

Figure 7: Temporal evolution of AIDS Idealized in Manaus - SIR model



Source: Author

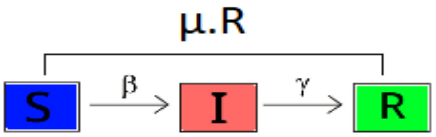
In the initial iterations, we have an increase in the number of those infected, which causes an increase in those recovered and a decrease in those who are susceptible. In this case, the percentage of infected reaches a little more than 20%, at its maximum value. We found that in iteration number 959, that is, in 96 days, we have the maximum share of infected people found. After 200 days, the population of infected people disappears, as they will be part of the group of those recovered. This group of those removed grows until it reaches approximately 89% of the population. A portion of approximately 11% remains susceptible after 150 days. See Figure (7).

Notice that we started in an SI model, where the entire population became infected, we migrated to a SIS model, in which the percentage of infected, despite being able to indicate a serious situation, stabilized, at a percentage of less than 100%, after a few days, and now, in the SIR model, the maximum share of infected people can be lower in relation to the SIS model, and after a few days, the infected population is extinct. With this, we continue in our evolution.

SIRS MODEL

Moving on to a next modification, let us consider that after being inserted into the class of those recovered, individuals are again susceptible. What we are dealing with from now on refers to the SIRS model, Figure (8). The differential system is represented in Equation (4).

Figure 8: Compartment diagram representing the SIRS model



Source: Author

Using the data already used initially, let's take a μ rate for immunity loss, very close to or equal to zero, with this, we will have the temporal evolution already presented previously, that is, we will have the SIR model. Which means, that we have to take values to μ closer to 1.

Now, if the infected individual manages to recover, acquiring immunity and after an interval of time, becomes susceptible again, that is, becomes vulnerable to the disease again, we can imagine that in the SIRS there will be a setback, worsening the situation obtained with the SIR.

$$\begin{cases} \frac{dS}{dt} = -\beta . S . I + \mu . R \\ \frac{dI}{dt} = \beta . S . I - \gamma . I \\ \frac{dR}{dt} = \gamma . I - \mu . R. \end{cases} \quad (4)$$

According to Luiz (2012, p.40), "in this model there are susceptible individuals who acquire the disease, becoming infected and, after recovery, do not acquire immunity, becoming susceptible again".

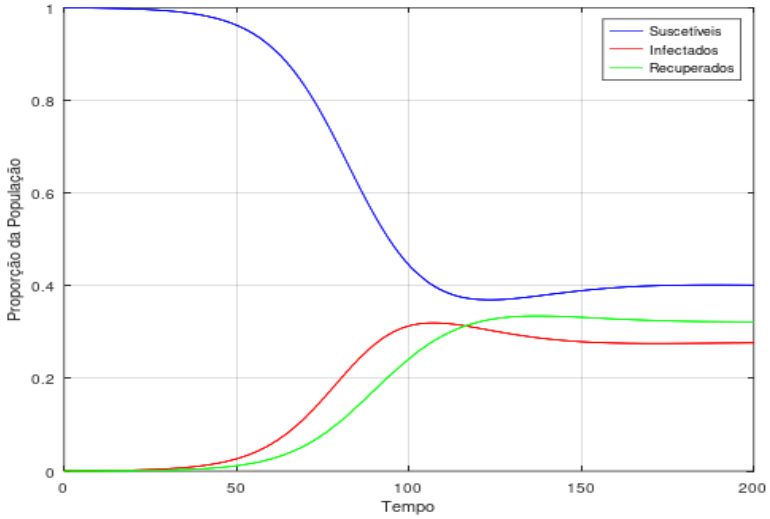
> As mentioned above, the big difference between SIRS and SIR is that, for SIRS, an individual can lose their immunity after the disease is cured, as in the case of tuberculosis and malaria. This new characteristic of reinfection can occur in two ways: either the infected individual, when cured, goes directly to the group of the susceptible, or the infected individual, when cured, goes to the group of the recovered, and a part of this group becomes susceptible again (SIRS model). (VIEIRA, 2016, p. 29).

We will assume a rate μ = 0.99, with the intention of analyzing the SIRS model, with this, we obtain a graph, referring to the SIS model, but containing the curve of the retrieved with very small values. Which means that if the immunity loss rate, is very close to zero, we get the SIR model and if it is very close to 1, we will return to the SIS model.

If we take μ = 0.05, we can see, analyzing the temporal evolution, with γ = 0.05816 and using the same contagion rate β = 0.1454, that the susceptible decrease, reaching a minimum point of 37%, approximately, and then will grow again, stabilizing at 40%. The infected, on the other hand, grow, reach a maximum point of approximately 32%, and then stabilize at 28%. The number of recovered individuals increased and surpassed those

infected between 113 and 120 days, stabilizing at approximately 32%, as can be seen in Figure (9).

Figure 9: Temporal evolution of AIDS Idealized in Manaus - SIRS model

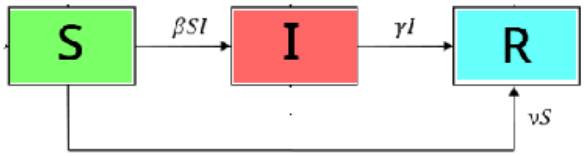


Source: Author

SIRV MODEL

To complete the process of evolution of epidemiological models for Idealized AIDS, we need to consider that there is a vaccine capable of immunizing the susceptible. Thus, a portion of these individuals will be immunized, so that none of them, after being vaccinated, will develop the disease. For this reason, we need a safe and effective vaccine. See Figure (10).

Figure 10: Compartment diagram representing the SIRV model.



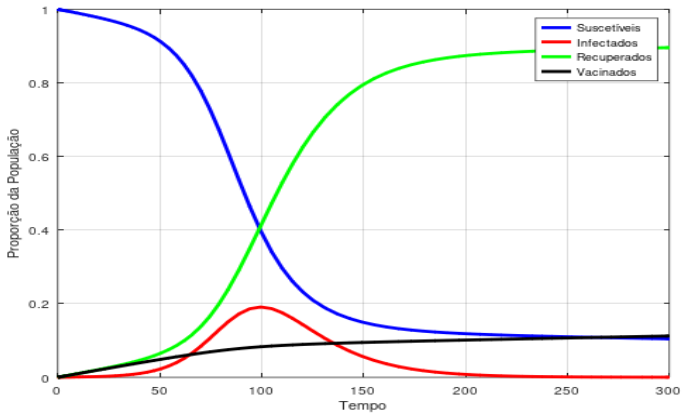
Source: Author

If we consider values very close to or equal to zero, for the v vaccination rate, we will return to the temporal evolution of the SIR model. Note the representative system of differential equations of the SIRV model in Equation (5).

$$\begin{cases} \frac{dS}{dt} = -\beta.S.I - v.S \\ \frac{dI}{dt} = \beta.S.I - \gamma.I \\ \frac{dR}{dt} = \gamma.I + v.S. \end{cases} \quad (5)$$

Let's assume that the vaccination rate is equal to 0.001, that is, we have 0.1% of the susceptible receiving the vaccine, and 99.9%, those who will not be vaccinated. Due to several factors, susceptible people can avoid vaccination. Based on the inequality in which the initial number of susceptible people is greater than the ratio between the rate of recovery and contagion, we can determine whether or not the disease spreads among the population. In this case, 0.99964 > 0.4, with this, the disease spreads among the population. With the intervention of the vaccine, we have 0.999.S< 0.4, which means S<0.4, which causes the infected to start decreasing in the iteration where the susceptible ones assume a value lower than 0.4. See Figure (11). This analysis is extremely important when it comes to this model.

Figure 11: Temporal evolution of AIDS Idealized in Manaus - SIRV model

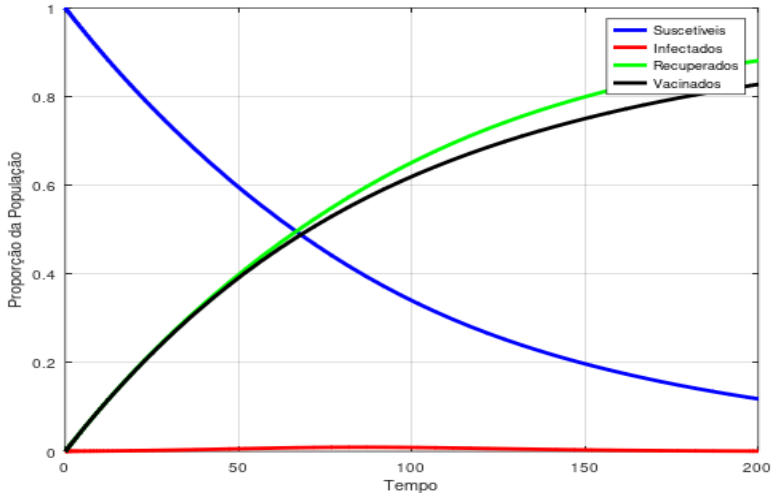


Source: Author

When the susceptible reach the percentage of approximately 40%, the infected reach the maximum point of approximately 19% and begin to decrease, until they do not exist. Those who have recovered are equal to those who are susceptible and then surpass. In iterations number 2496 to 2500, the vaccinated grow and stabilize at approximately 11%, as well as the susceptible, who decrease and stabilize in the same range. Those recovered stabilize at approximately 89%.

We can verify, by observing the temporal evolutions (Figure (12)), that as the vaccination rate increases, the maximum number of infected people decreases, until there are no more infected people and the red curve is fixed on the axis of the abscissas. Let's take a vaccination rate equal to 0.01.

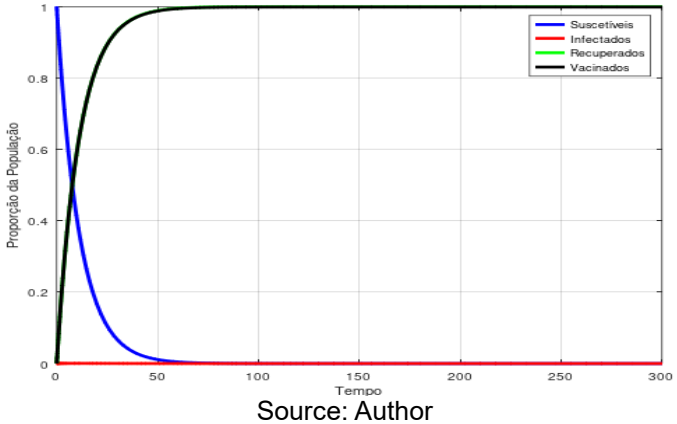
Figure 12: Temporal evolution of AIDS Idealized in Manaus - SIRV model



Source: Author

Now, let's imagine that 9% of the susceptible and recovered have been vaccinated. We have that the percentage of unvaccinated is equal to 91%, so S<0.44, approximately. For this reason, the intervention is carried out at the beginning and there is no growth of those infected. After 50 days, the entire population is vaccinated and there are no more susceptible individuals.

Figure 13: Temporal evolution of AIDS Idealized in Manaus - SIRV model



Source: Author

Therefore, in this last simulation of evolution, for the SIRV model, the disease spreads among the population. However, in the iteration in which the number of susceptible people is less than or equal to the ratio between the recovery and contagion rates, divided by the share of the unvaccinated, the infected begin to decrease until they disappear. For this to happen more quickly, the share of unvaccinated people needs to be as small as possible. Therefore, it is expected that the vaccination rate will be close to or equal to 1, which will mean that most or all of the population is already vaccinated, with this, the number of infected people will decrease as soon as possible. See Figure (13).

FINDINGS

In our evolution to AIDS in Manaus, we began to analyze the reality, where all susceptible people become infected over time. As we increased the contagion rate, we could see that the contamination of the entire population occurred more quickly. In addition, we observed that, while the number of infected people increased until it stabilized at 100%, when it reached the entire population, the susceptible ones decreased until they did not exist. When we introduced a recovery rate, we saw that this stabilization of the infected began to involve only a portion of the population, and the higher this rate, this portion began to decrease. As a result, the susceptible ones decreased, but continued to exist, stabilizing, and could even exceed the number of infected. From the SIS model, the coefficient R 0 emergesβ , which will act as an indicator, that some measure must be taken to contain the spread.

Now, when we consider that the recovery rate made a portion of the infected immunized, we were able to further reduce the number of infected, because despite the possibility of growth of this class, they reached a maximum percentage and began to decrease, until they did not exist. Through this modification, there was the appearance of the category of recovered, where the former infected become immune. In addition, the higher the recovery rate, the lower the maximum share of infected people. When we consider that this immunization had a limited duration, we worsened the situation obtained with the SIR model, as we decreased the number of recovered and increased the number of infected and susceptible.

To conclude the evolution, we introduced a vaccination, which managed to return to the situation obtained with the SIR model, causing a decrease in the maximum number of infected people. We then have, with the SIRV model, a new category, that of the vaccinated. We observed that the higher the rate of vaccinated, the less infected we will have, since the more susceptible will be vaccinated. For this to happen more quickly, the share of unvaccinated people needs to be as small as possible. As a result, there may be no one infected. Therefore, it is expected that the vaccination rate will always be close to or equal to 1, in the case of Idealized AIDS.

CHALLENGES IN IMPLEMENTING A VACCINE

In view of the purpose of this article, we emphasize the fact that we need vaccines that can be produced and distributed throughout Brazil. Although there are antiretroviral treatments, which allow those infected to have a healthy life, many researchers and medical technology companies work incessantly in search of an immunizer that can bring safety and

is effective in the treatment and recovery of these individuals. However, there are records of several unsuccessful attempts to search for this vaccine. Therefore, obtaining a vaccine that immunizes the population is still a great challenge.

> The challenges that hinder the creation and use of the immunizer on a large scale involve the genetic variability of the virus, since HIV type 1 (HIV 1), the main cause of AIDS, is currently subdivided into three groups. HIV-2 is associated with infections in West-Central Africa and Europe. (CECCON. et.al, 2024, p.1).

All people diagnosed with HIV should immediately start treatment with antiretrovirals. Thus, the virus will be prevented from replicating inside the cell, which will prevent a decrease in immunity and the worsening of symptoms. But this treatment, after decades of epidemics, is no longer enough.

> According to Turan et.al. (2016 apud CECCON et.al, 2024, p. 3), scientific research on the AIDS vaccine is underway in different countries, but most of it is focused on the clinical and biomedical perspective, generating gaps in the production of knowledge on the ethical, political, and social perspective. Thus, although it is a measure of collective interest, the topic is little debated publicly, which hinders social participation and tension to transform this agenda into political interest. This fact may be due to the stigmatization of people living with HIV, making their demands not of interest to society, being silenced in the communication vehicles that make up the Brazilian media.

We need to adopt a critical perspective regarding the real need for a vaccine, even if we are part of the group of the susceptible, understanding that scientists use technical and biomedical terms, which many will not be able to understand, but we all have a universal language, which is the understanding of the importance of creating and implementing a vaccine, which can be widely available to the entire population.

> AIDS, four decades after the beginning of the epidemic, is still an important public health problem in Brazil, with high incidence and mortality rates. Prevention strategies focus on condoms and pre- and post-exposure prophylaxis, and no vaccines have been implemented so far. There was little media communication about the AIDS vaccine, making it a neglected topic that is little explored in the public debate. The media discourses pointed to communication that was difficult to understand, with biomedical terminologies; they focused on the impossibilities of scientific research in the discovery of vaccines; and portrayed the respondents as "guinea pigs", a metaphor that names groups vulnerable to drug experiments, which reinforces discrimination and prejudice. The speeches point to the need for public debate on the AIDS vaccine, and the Brazilian media has a strategic role in stimulating the production of immunobiologicals. (CECCON. et.al, 2024, p.1).

If it is necessary to carry out tests on humans, this should be done with full responsibility, not turning these subjects into "guinea pigs", since many of them may be part of vulnerable groups and capable of collaborating with drug tests, reinforcing decriminalization and prejudice. The AIDS vaccine is an important strategy for the HIV epidemic, so it should be widely discussed in Brazilian society.

CONCLUSION

Mathematical modeling plays a crucial role in understanding data and predicting the evolution over time of infectious diseases, such as AIDS. Through modeling, it is possible to create a clear and organized view of situations in the present or in the future, caused by diseases, which facilitates the understanding and analysis of information. The models use systems of nonlinear ordinary differential equations to describe the dynamics of the propagation. These models help to estimate the number of cases, per period and to evaluate control strategies.

What is expected with this article is the construction of a proposal for the evolution of AIDS, using some of the basic models that are being applied in the study of some diseases. These studies become essential in the attempt to predict scenarios, analyzing a supposed evolution to this disease, which would allow the SI model to be replaced by a new model, where those infected do not represent the entire population. Therefore, we hope that new treatments and vaccines for AIDS are tested and can be implemented in the future, benefiting the Brazilian and world population, which longs for investigations and studies.

In addition, we seek to contribute to the contextualization of Mathematics at a higher level, through the use of a real problem. In addition to awakening the desire to learn how to model, enriching the state of the art. This search for solutions to real situations brings numerous benefits to society. It is hoped that these data can contribute to the reflection on the theme, as well as to the orientation or direction of actions. We hope to awaken the desire to study these and other models involved, as well as to contribute to the investigation of these processes.

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REFERENCES

1. Beck, C. (2014). Análise de sobrevida e perfil epidemiológico de casos de AIDS em Porto Alegre/RS: Limitações e potencialidades da vigilância epidemiológica [Unpublished master's dissertation]. Universidade Federal do Rio Grande do Sul.

2. Campany, L. N. da S., et al. (2021). HIV/AIDS no Brasil: Feminização da epidemia em análise. Revista Bioética, 29(2), 351–361. https://doi.org/10.1590/1983-80422021292503

3. Ceccon, R. F., et al. (2024). Vacina contra a AIDS na mídia brasileira: Análise crítica do discurso. Universidade Federal de Santa Catarina. https://repositorio.ufsc.br/handle/123456789/256789

4. Lopes, L. E. da S., & Prata, R. A. C. (2019). Modelo epidemiológico para a AIDS em Manaus-AM com a solução interativa fuzzy. Biomatemática, 29, 69–80.

5. Luiz, M. H. R. (2012). Modelos matemáticos em epidemiologia [Unpublished master's dissertation]. Universidade Estadual Paulista.

6. Ministério da Saúde. (2019). AIDS/HIV: O que é, causas, sintomas, diagnóstico, tratamento e prevenção. http://portalms.saude.gov.br

7. Quadros, A. S. (2013). Modelos epidemiológicos para propagação de informação [Unpublished master's dissertation]. Universidade Federal do Rio de Janeiro.

8. Teles, P. (2020). Modelos compartimentais e aplicações. Revista Ciência Elementar, 8(2), 1–10.

9. Turan, B., Budhwani, H., Fazeli, P. L., Browning, W. R., Raper, J. L., Mugavero, M. J., & Turan, J. M. (2017). How does stigma affect people living with HIV? The mediating roles of internalized and anticipated HIV stigma in the effects of perceived community stigma on health and psychosocial outcomes. AIDS and Behavior, 21(1), 283–291. https://doi.org/10.1007/s10461-016-1451-5

10. Vieira, A. de O. (2016). Estudo sobre modelos matemáticos aplicados à epidemiologia: Modelo SIR, SIR com vacinação e SIRS [Unpublished undergraduate thesis]. Instituto Federal de São Paulo.