APPLICATION OF CHAOS THEORY TO DIFFERENT MIXED SLOPE GEOMETRIES

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ABSTRACT

Mass movements are natural phenomena that frequently occur on slopes, especially in mountainous and densely populated regions, exhibiting destructive behaviors, with the loss of life and destruction of material goods. Therefore, the in-depth study of slope stability is extremely important in geotechnical engineering. The intricate nature of these phenomena makes it extremely difficult to predict them and to apply conventional models and analyses. The Mathematical Chaos Theory emerges as a promising tool to understand the supposedly unpredictable and sometimes illogical behavior of certain physical phenomena, characterized by the interaction of many agents, in a cooperative process, in which the behavior of the whole is not reduced to that of its constituent parts. This leads to marked nonlinearities and sensitivity to initial conditions, which makes the search for (closed) mathematical analytical solutions difficult, if not impossible. This work aims to deepen the application of Chaos Theory to mass movements on rock slopes of mixed surfaces (concave and convex), aiming at the analysis of the stability and fall of blocks of irregular geometry, their dynamic evolution, sensitivity to initial conditions, among other related aspects, in the wake of the research inaugurated by Ignácio (2019). The results of this research showed that the relative arrangement of the concave and convex regions in the constitution of the profile of natural slopes has a marked influence on the response of the rock blocks located downhill. In more detail, convex regions close to the release of the blocks (top of the slope) give rise to evolution dynamics adequately described by non-Gaussian statistics (extended q-exponentials). On the other hand, in the situation where those convex regions are closer to the base level of the natural slopes, their dynamics are more appropriately described by a statistical distribution constituted by the linear combination of the Gaussian and extended q-exponential distributions.

Keywords: Chaos theory; rock slopes; nonlinear systems; mass movements.

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INTRODUCTION

Slope stability on rocky slopes is an extremely important topic in geotechnical engineering, since the occurrence of mass movements, such as falls and block rolling, has an enormous destructive potential. Due to the complexity and unpredictability of these phenomena, their analysis becomes quite difficult, which has been challenging traditional approaches, however, their understanding is fundamental in mitigating the associated risks and adopting security measures.

Rock masses are made up of intact rock material and discontinuity surfaces, often presenting high heterogeneity. Its behavior is a function of a series of factors – resistance and deformability of the rock matrix and discontinuities, weathered behavior of the material, etc. – which makes it difficult to identify regularities and reiterations in its movements, which are often unpredictable and difficult to identify by traditional methods. It is in this context that Chaos Theory emerges as a promising tool for investigating and understanding these complex phenomena.

This mathematical theory deals with physical phenomena, characterized by the interaction of several agents, in a common process of cooperation, in which the behavior of the whole is not reduced to the behavior of its constituent parts. Accentuated nonlinearities, extreme sensitivity to initial conditions, among other factors, make the phenomenon difficult to analyze, in which the search for mathematical analytical solutions becomes difficult, if not impossible.

Applied to geotechnical phenomena, especially those related to the instability of natural slopes in soil and rock, the theory has produced promising results, which are extremely important in the understanding and mitigation of these complex natural phenomena.

This research aims to deepen the application of chaos theory to mass movements on rock slopes with concave and convex geometric profiles, analyzing the shape of the slopes and the different types of falls and bearings of irregular rock blocks that may occur, as well as their dynamic evolution, sensitivity to initial conditions, among other aspects of interest, in the wake of the line of research inaugurated by Ignácio (2019).

Ignácio (2019) observed a huge variability of trajectory-range probability distributions for different geometries of concave and convex slopes, concluding that the results obtained in convex profiles provide Gaussian trajectory-range probability distributions, while in concave sections, with or without smooth convex sections, non-Gaussian probability distributions, such as extended q-exponentials, present a better fit of the points than those of the Gaussian probability distributions.

Thus, this work aims to contribute to a deeper knowledge of these complex phenomena, resulting in more effective strategies for prevention and mitigation of geotechnical risks.

LITERATURE REVIEW

BASIC CONCEPTS OF FALLING ROCK BLOCKS AND CHAOS THEORY

Rock slopes are made up of intact rock and discontinuities, these geological features are described as faults, joints, beddling planes and fissures. The overall strength of a massif depends on the strength of the rock and the characteristics of these discontinuities, whose relative participation in the overall stability is still difficult to quantify (AZEVEDO and MARQUES, 2002).

From a geotechnical point of view, the studies allow a greater understanding of the triggering mechanisms of such movements, whether they are landslides, falling blocks, among others, which are very common on slopes in rocky and/or earthy materials (GUIDICINI and NIEBLE, 2019).

Falling blocks, in particular, are characterized by rapid and often unpredictable movements, in which rock fragments break off steep slopes, moving in free fall and sometimes rolling or sliding. In general, these events occur suddenly, without previous signs of movement, and reach high speeds, developing high kinetic energies during the displacement, which makes them especially dangerous and difficult to anticipate or control (IGNACIO, 2019).

This unpredictability leads to the need for models that contemplate non-linear dynamic behaviors that are sensitive to initial conditions, characteristics often observed in chaotic systems. (MENEZES FILHO, 2003).

In the context of chaos theory, nonlinear dynamical systems exhibit extreme sensitivity to initial conditions. This characteristic implies that small variations in the initial state of the system can generate significantly different trajectories over time. This sensitivity is related to the presence of strange attractors in phase space, structures that define the evolution of the system in an apparently disordered way, but which still follow specific mathematical patterns (MANDELBROT, 1977).

To describe the degree of internal disorder of such systems, the concept of *entropy* is applied, understood as a macroscopic indicator of the complexity and dynamic behavior resulting from microscopic interactions. In the case of extensive systems with short-range interactions, Boltzmann-Gibbs entropy is widely used. This entropy is additive and reaches its maximum value in the thermal equilibrium of the system. The probability distribution that maximizes it is the Gaussian (PEDRON, 1999), given by Equation (1).

$$p(x) = a\left[e^{\left[-\left(\frac{x-c}{b}\right)^2\right]}\right] \quad (1)$$

where *a, b,* and *c* are adjustment parameters.

However, many physical systems, especially those with long-range interactions or complex spatial and temporal structures, do not fit this classical formulation. For these cases, entropy is not an additive property, and the conventional approach is no longer valid (MENEZES FILHO, 2003).

Tsallis (1988) then proposed a generalization of Boltzmann-Gibbs entropy, allowing a new possibility of dealing with anomalous, non-additive systems, said to be weakly chaotic or complex (LIMA, 2021). This can be written as shown in Equation (2), where , , , and are parameters of fit.$a'b'c'q\delta$

$$p(x) = a'\left[1-(1-q)\left(\frac{x'-c'}{b'}\right)^{\frac{2}{\delta}}\right]^{\frac{1}{1-q}} \quad (2)$$

This distribution has been successfully applied in the statistical description of the phenomenon of falling rock blocks, especially those that reach the foot of the natural slopes down the slope (IGNÁCIO, 2019; RIBEIRO, 2020; ARAÚJO, 2021).

METHOD OF ANALYSIS

The simulations were carried out based on case C2 of Ignácio's research (2019). The geometry of the slope was systematically modified, varying the position of the convex and concave surfaces along the slope, in order to analyze the influence of these variations on the behavior of falling blocks.

The rocky slope was entirely modeled by facoidal gneiss, and the configurations of the blocks were also provided by Ignacio (2019), and the use of Large Irregular Blocks (BIG) was defined in all analyses.

The results of the distribution of blocks at each point of the surface are obtained by the software by means of histogram and the data can be exported in spreadsheets.

Ferreira's master's thesis (2024) summarizes the studies developed by PGECIV/UERJ researchers, focused on the analysis of the fall of rock blocks, highlighting the effectiveness of numerical simulations and probabilistic analyses based on the theory of chaotic and complex systems. The results proved the usefulness of this approach to understand and predict this phenomenon.

The parameters of the phasoidal gneiss constituting the slope and the blocks were defined in Menezes Filho (1993). In more detail, laboratory uniaxial axisymmetric compression tests were performed on several samples of phachoidal gneiss, with weathering levels ranging from healthy to highly altered. Resistance and deformability parameters were determined and used for a better characterization of the rock under study.

The data related to the normal and tangential restitution coefficients, as well as those related to dynamic and rolling frictions, were those used by Ignácio (2019), as shown in Table 1.

Table 1- Parameters for the geotechnical modeling of facoidal gneiss slopes (Adapted from Ignácio, 2019)

Rock material	Normal Refund Coefficient (CRn)	Tangential Refund Coefficient (CRt)	Dynamic friction	Rolling friction
Phasoidal gneiss	0,35	0,85	0,5	0,15

Based on the information provided by Ignácio (2019), the BIG block with irregular geometry was chosen, with the following characteristics: mass of 7,408.8 kg, specific weight of 27 kN/m³ and approximate size of 1.40 meters.

The choice of the C2 profile was due to its better suitability for the purposes of the research, i.e., its refined geometry and convex and concave surfaces closer to reality.

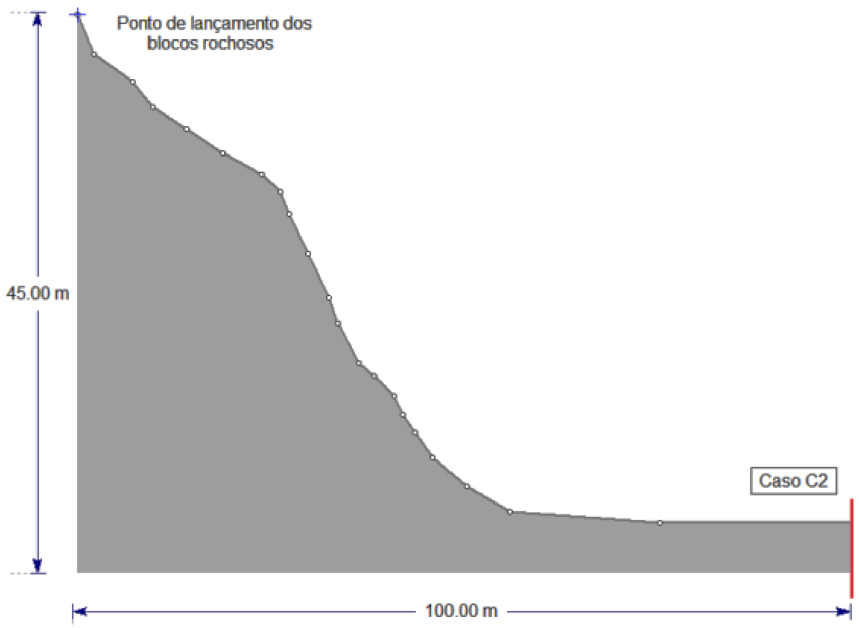

Figure 1 Slope studied in case C2 by Ignácio (2019)

In order to investigate the mutual influence of the convex and concave geometries on the profile of the rocky slope, it was decided to monitor the most prominent point in the convex part, called Y (Figure 2), located at 30.75 meters above the base of the model. From now on, this surface will be named as Base Scenario.

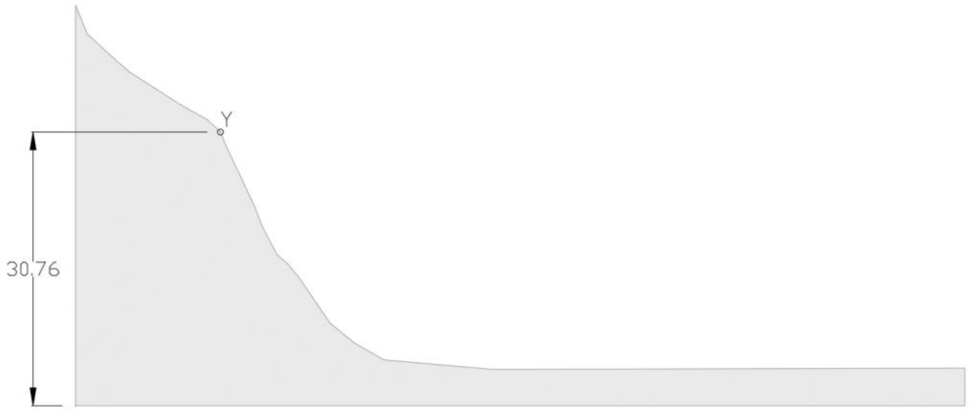

Figure 2 Base Scenario: Slope Defined

Monitoring this point served to change the location of the convex part along the slope surface. More precisely, the variation of point Y occurred along a straight line with an average slope of the slope surface, with an angle of 48°. This line was divided every meter, with the 19th distribution corresponding to point Y related to the base configuration. The other scenarios were modeled according to the division adopted and presented later.

The configurations adopted in the present research were similar to those used by Ignácio (2019), and are presented in Table 2.

Table 2 - Project Configuration

Project Settings		
General Settings	Engine	Rigid Body
	Units	Metric (m, kg, kJ)
	Rock throw mode	Number of rocks controlled by seeder
Engine Conditions	Use tangential CRSP damping	Yes
	Maximum steps per rock	20000
	Normal velocity cutoff (m/s)	0.1
	Stopped velocity cutoff (m/s)	0.1
	Maximum timestep(s)	0.01
	Switch velocity (m/s)	-1e-09
Random Number Generation	Sampling method	Monte-Carlo
	Material Properties Sampling	Per segment
	Random seed	Pseudo-random seed: 12345234

The unstable blocks were considered as rigid bodies and, in the probability configuration, it was decided to use the Monte Carlo sampling method, due to its better adaptation for the estimation of the results of complex functions, dealing with probabilistic.

In the Mechanism Conditions tab, the speed parameters were used as suggested by the RockFall2 software, version 8.023, made available in October 2023 by Rocscience. Although 100,000 blocks were launched, as adopted in previous research, many simulations resulted in only 20,000 effective launches, which led to less robust statistics.

The launching point of the blocks, called seeders, was inserted in the highest place on the slope.

For data treatment, the *Grapher* software (version 16.2.354, Golden Software, May 2020) was used to generate 2D graphs. The experimental data were adjusted using the Gaussian and extended q-exponential probability distributions, represented by Equations (1) and (2), respectively, whose quality of adjustments was evaluated by the correlation coefficient R.

RESULTS

PROBABILISTIC DISTRIBUTIONS

Two main probability distributions were used in the treatment of experimental data related to the position and number of blocks located at the foot of the slopes. The first was the Gaussian distribution, used for the analysis of strongly chaotic systems, with exponentially rapid dispersion between initially very close trajectories. The second was extended q-exponential, applied to weakly chaotic or complex systems, with slower dispersion between initially close trajectories.

The Gaussian and extended q-exponential distribution will be represented by the colors blue and red, respectively. The analyzed scenarios were named as follows: Base Scenario (corresponding to Case C2 mentioned above), Scenario 01, Scenario 02, Scenario 03, Scenario 04 and Scenario 05.

Base case

Represented in Ignácio's thesis (2019) as case C2, it gave rise to the other scenarios studied in this research.

Profile data:

- Observation point: the most prominent point in the convex section, previously named Y, is located 30.76 meters from the base of the model
- Cross section: 45 meters high
- Length: 100 meters

Results shown in Figure 3.

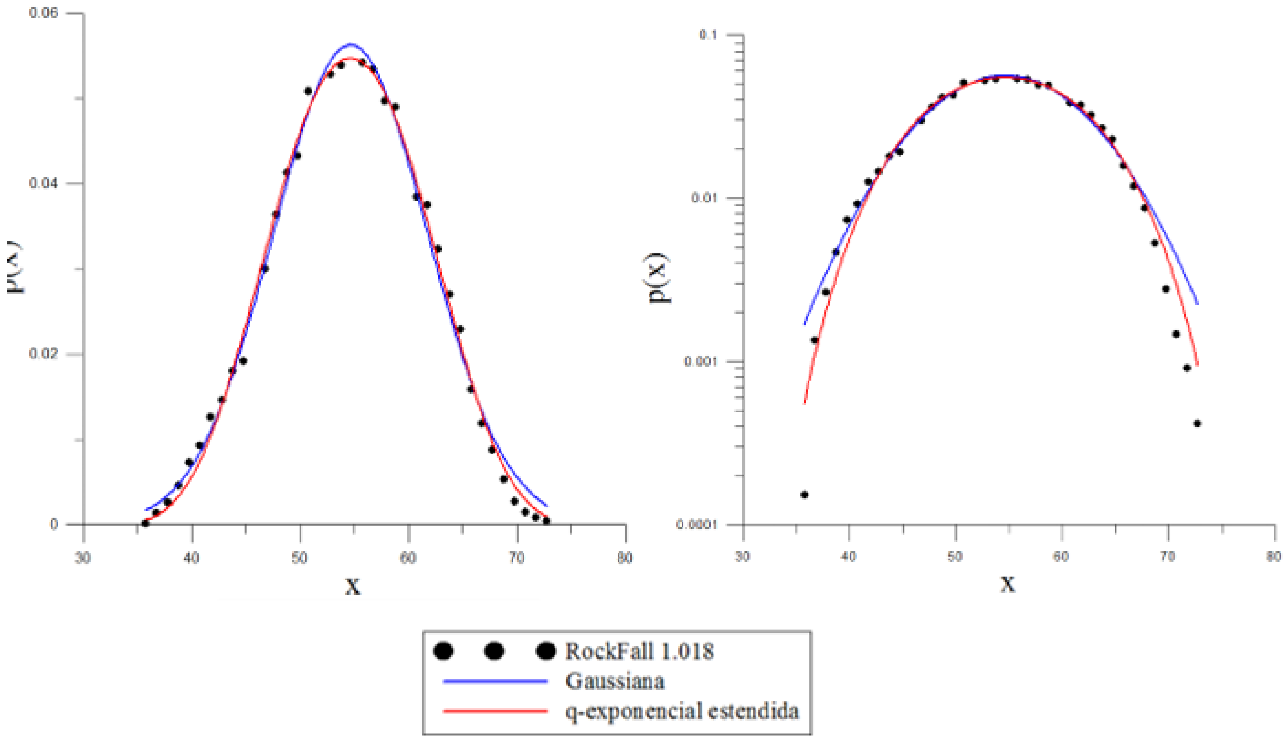

Figure 3 Graph p(x) versus x and Graph log(p(x)) versus x - Base Scenario

Scenario 01

In this case, the convex surface is located adjacent to the top and the concave surface predominates in the rest of the slope.

Profile data:

- Observation point: the most prominent point in the convex section, previously named Y, is located 34.46 meters from the base of the model
- Cross section: 45 meters high
- Length: 100 meters

Results shown in Figure 4.

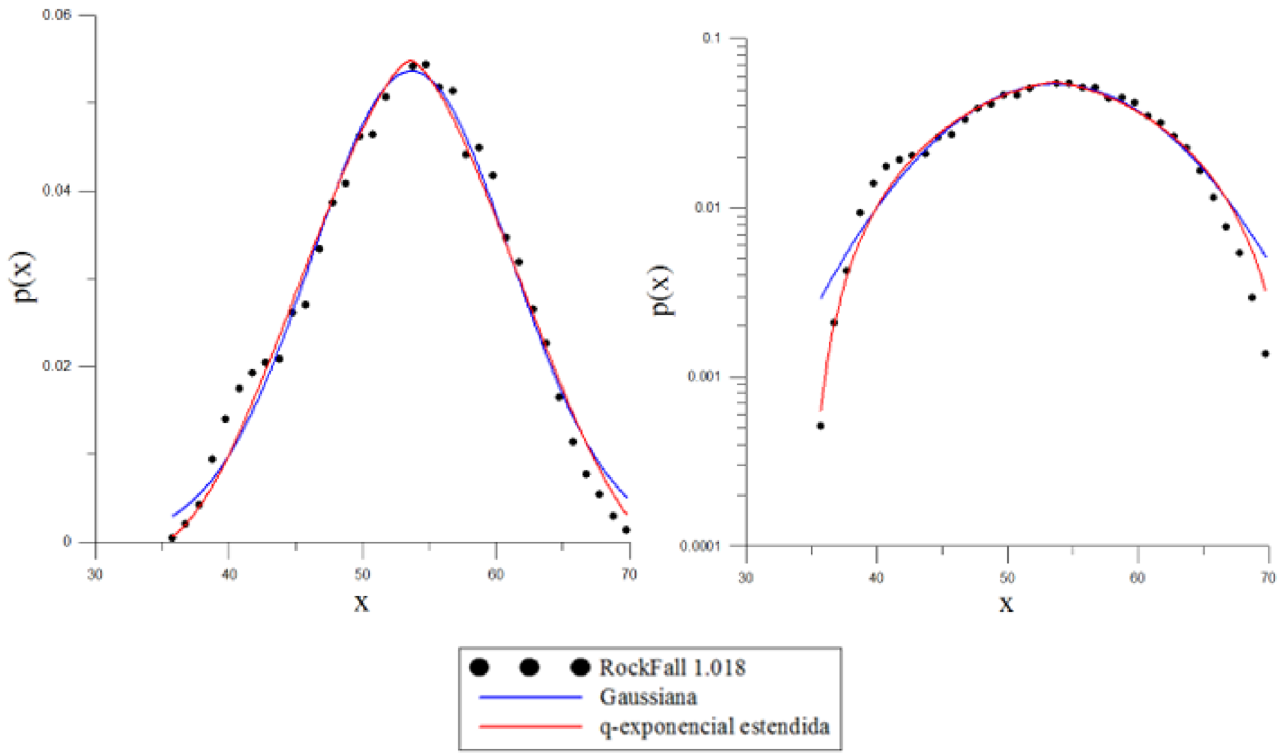

Figure 4 Graph p(x) versus x and Graph log(p(x)) versus x - Scenario 01

Scenario 02

In this case, the convex surface is close to the top of the slope, and the concave surface predominates in the rest of the slope.

Profile data:

- Observation point: the most prominent point in the convex section, previously named Y, is located 32.61 meters from the base of the model
- Cross section: 45 meters high
- Length: 100 meters

Results presented in Figure 5.

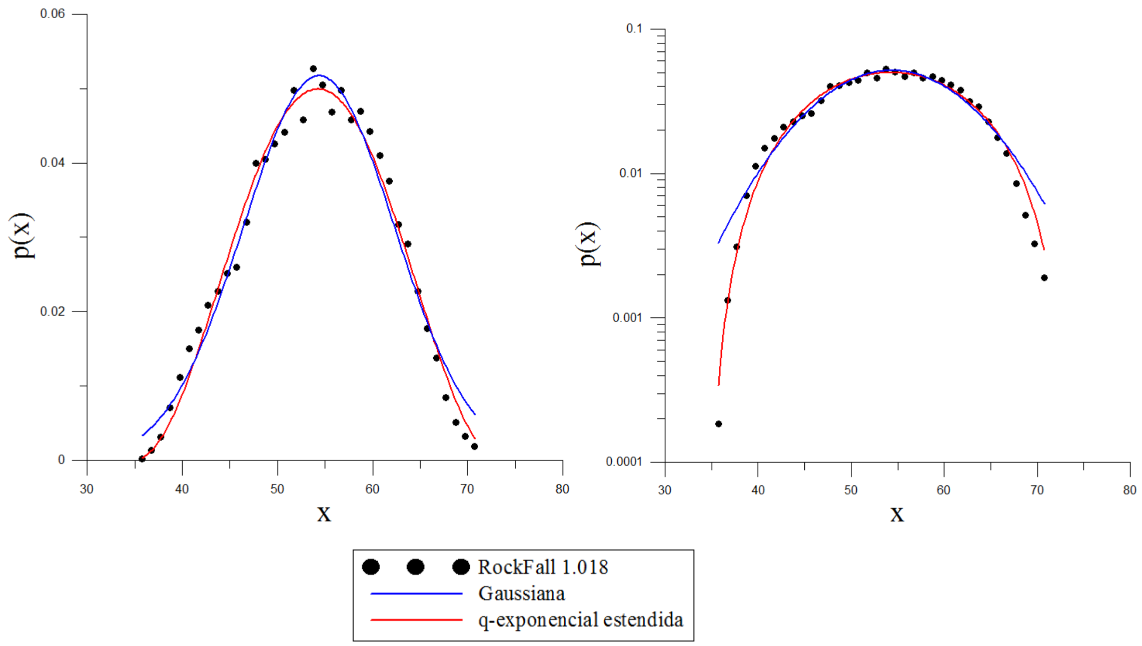

Figure 5 Graph p(x) versus x and Graph log(p(x)) versus x - Scenario 02

Scenario 03

In this case, the convex surface is located approximately halfway up the slope, and the concave surface predominates above the Y point and near the foot of the slope.

Profile data:

- Observation point: the most protruding point in the convex section, previously named Y, is located 23.37 meters from the base of the model
- Cross section: 45 meters high
- Length: 100 meters

Results presented in Figure 6.

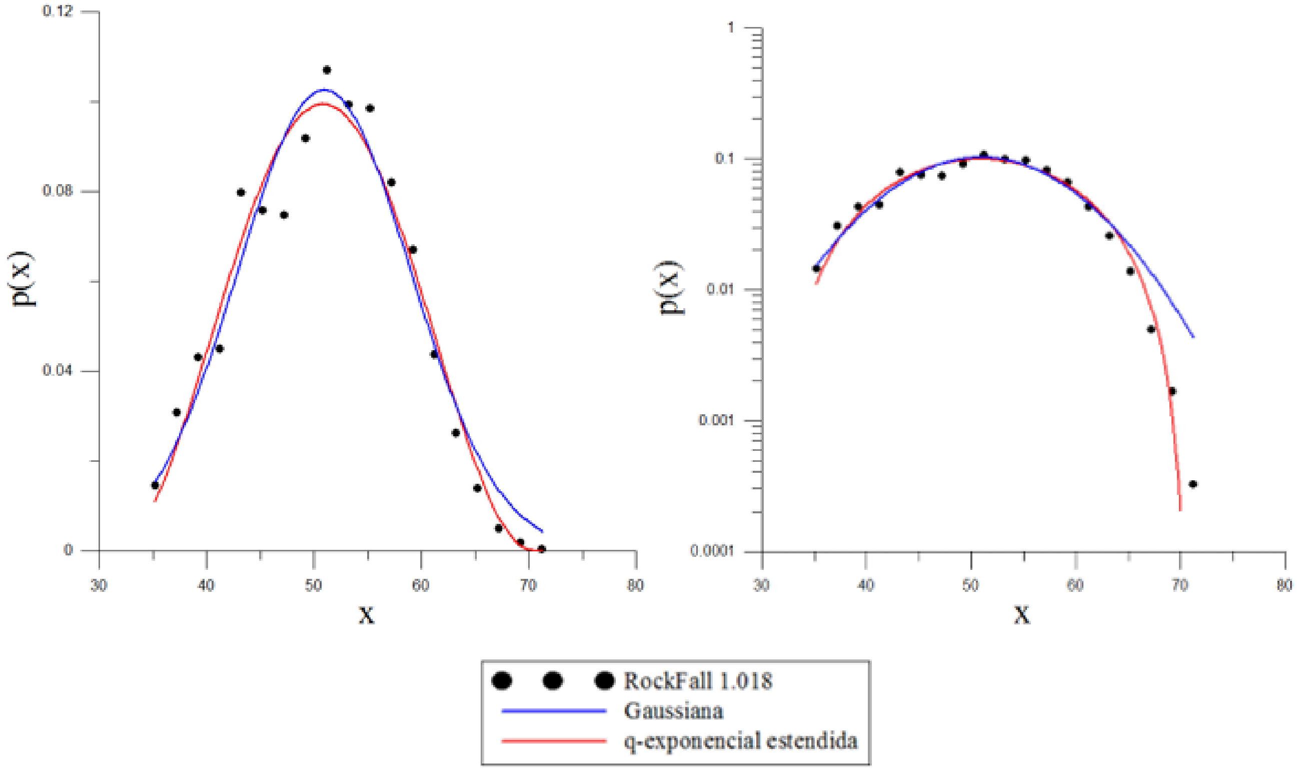

Figure 6 Graph p(x) versus x and Graph log(p(x)) versus x - Scenario 03

Scenario 04

In this case, the convex surface is located near the base of the slope and the concave surface predominates above the Y point.

Profile data:

- Observation point: the most protruding point in the convex section, previously named Y, is located 21.15 meters from the base of the model
- Cross section: 45 meters high
- Length: 100 meters

Results presented in Figure 7.

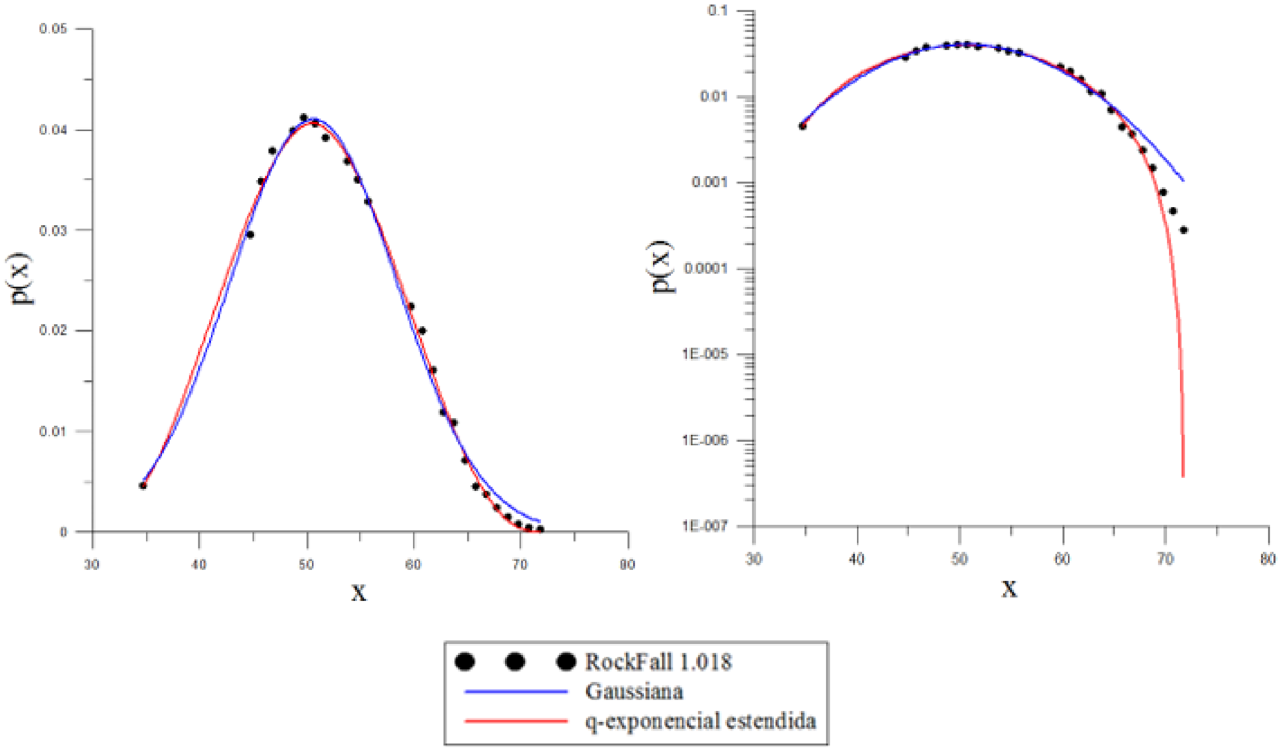

Figure 7 Graph p(x) versus x and Graph log(p(x)) versus x - Scenario 04

Scenario 05

In this case, the convex surface is located adjacent to the base of the slope and the concave surface predominates above the Y point.

Profile data:

- Observation point: the most prominent point in the convex section, previously named Y, is located 10.80 meters from the base of the model
- Cross section: 45 meters high
- Length: 100 meters

Results presented in Figure 8.

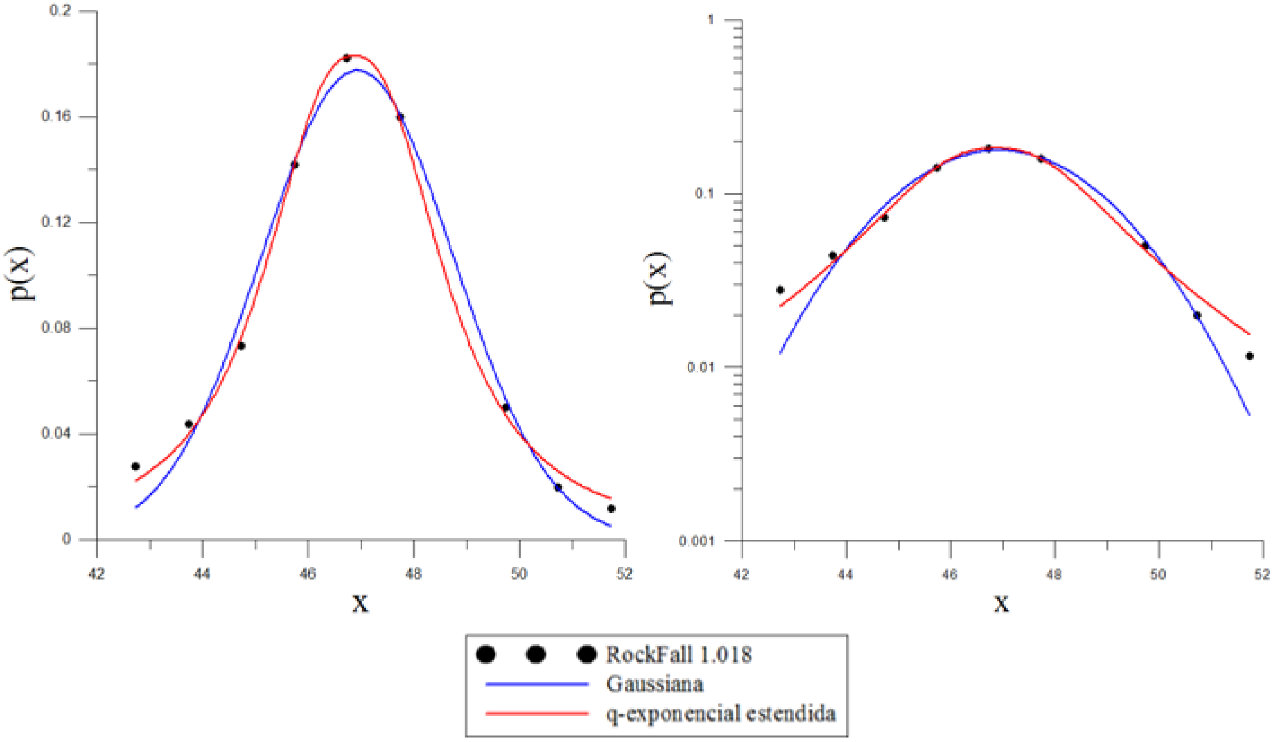

Figure 8 Graph p(x) versus x and Graph log(p(x)) versus x - Scenario 05

Considerations

Based on the graphs presented in the previous item, the following parameters for the adjustment of the Gaussian and extended q-exponential distributions and the correlation coefficients were obtained (Table 3). It should be noted that the adjustment parameters are presented in Ferreira (2024).

Table 3 Results obtained from the parameters used in the distributions

Scenario	Extended Q-Exponential		Gaussian	
	Parameters	Values found	Parameters	Values found
Base	q δ Correlation coefficient	0,8352076 0,9501157 0,9976617	Correlation coefficient	0,9961701
01	q δ Correlation coefficient	0,4733892 1,3069695 0,9912835	Correlation coefficient	0,9898382
02	q δ Correlation coefficient	0,5817001 0,9778995 0,9929233	Correlation coefficient	0,9876744
03	q δ Correlation coefficient	0,5620416 0,9993509 0,9818190	Correlation coefficient	0,9777841
04	q δ Correlation coefficient	0,59 1,11 0,998707	Correlation coefficient	0,997230

Scenario	Extended Q-Exponential		Gaussian	
	Parameters	Values found	Parameters	Values found
05	q	1,99		
	δ	0,81		
	Correlation coefficient	0,9984824	Correlation coefficient	0,9928663

The observation of the graphs and the parameters resulting from the adjustment of the curves suggest that:

In the base scenario, the probability distribution that best suited the experimental data was the extended q-exponential - especially in the tail region (Graph log(p(x)) versus x in Figure 3) - with a correlation coefficient of 0.9976617, when compared to the slightly lower value of 0.9961701, referring to the Gaussian distribution. The parameters q and δ showed values of 0.8352076 and 0.9501157, respectively, which indicates their proximity to the value 1, especially the parameter δ, constituting a distribution close to the q-Gaussian, whose statistics are quite common in weakly chaotic or complex systems.

In scenario 01, it is observed again that the extended q-exponential presented the best fit - especially in the tail region (Graph log(p(x)) versus x in Figure 4) - with a correlation coefficient of 0.9912835, slightly higher when compared to the value 0.9898382, referring to the Gaussian distribution. The parameters q and δ showed values of 0.4733892 and 1.3069695, respectively, suggesting a weakly chaotic behavior in the dynamics of destabilization of the rock blocks.

In scenario 02, the same trend follows: the extended q-exponential distribution showed better adherence to the data, especially in the tail region (Graph log(p(x)) versus x in Figure 5) - with a correlation coefficient of 0.9929233, higher than that of the Gaussian (0.9876744). The values of q = 0.5817001 and δ = 0.9778995 also point to weakly chaotic behavior, with the parameter δ close to unity — a recurring feature in the statistics of complex systems.

In Scenario 03, although the extended q-exponential continues to be the distribution that best fits the data (coefficient of 0.9818190, against 0.9777841 of the Gaussian), the adjustment of the tail region is only approximate (Graph log(p(x)) versus x of Figure 6). The parameters q = 0.5620416 and δ = 0.9993509 indicate a weakly chaotic behavior, with δ very close to 1. This strong approximation may be the cause of the imperfect fit in the tail, an issue that will be taken up again in item 3.6, which deals with the extended and Gaussian q-exponential joint distribution.

Scenario 04 presents a similar behavior to the previous one: the extended q-exponential fits better to the experimental data, especially in the tail (Graph log(p(x)) versus x in Figure 7), with a correlation coefficient of 0.998707, higher than the 0.997230 of the Gaussian distribution. The parameters q = 0.59 and δ = 1.11 indicate, again, a weakly chaotic behavior. The proximity of the δ parameter to the unit reinforces the observation of an approximate adjustment at the tail.

Finally, in Scenario 05, the same trend observed in Scenarios 03 and 04 is verified: the extended q-exponential adjusts better to the experimental data (coefficient of 0.9984824, compared to 0.9928663 of the Gaussian), but the adjustment in the tail region remains only approximate (Graph log(p(x)) versus x in Figure 8). The values of the parameters q = 1.99 and δ = 0.81 maintain the indication of a weakly chaotic behavior of the destabilization dynamics. As in the previous cases, this limitation will be deepened in item 4.6, dedicated to the analysis of the extended and Gaussian q-exponential joint distribution.

VARIATION OF ENTROPIC PARAMETERS Q AND Δ WITH THE POSITION OF THE MOST PROMINENT CONVEX POINT Y

Figure 9 shows the variation of the parameters q and δ with the height of the base of the slope in relation to the most prominent point of the convex section.

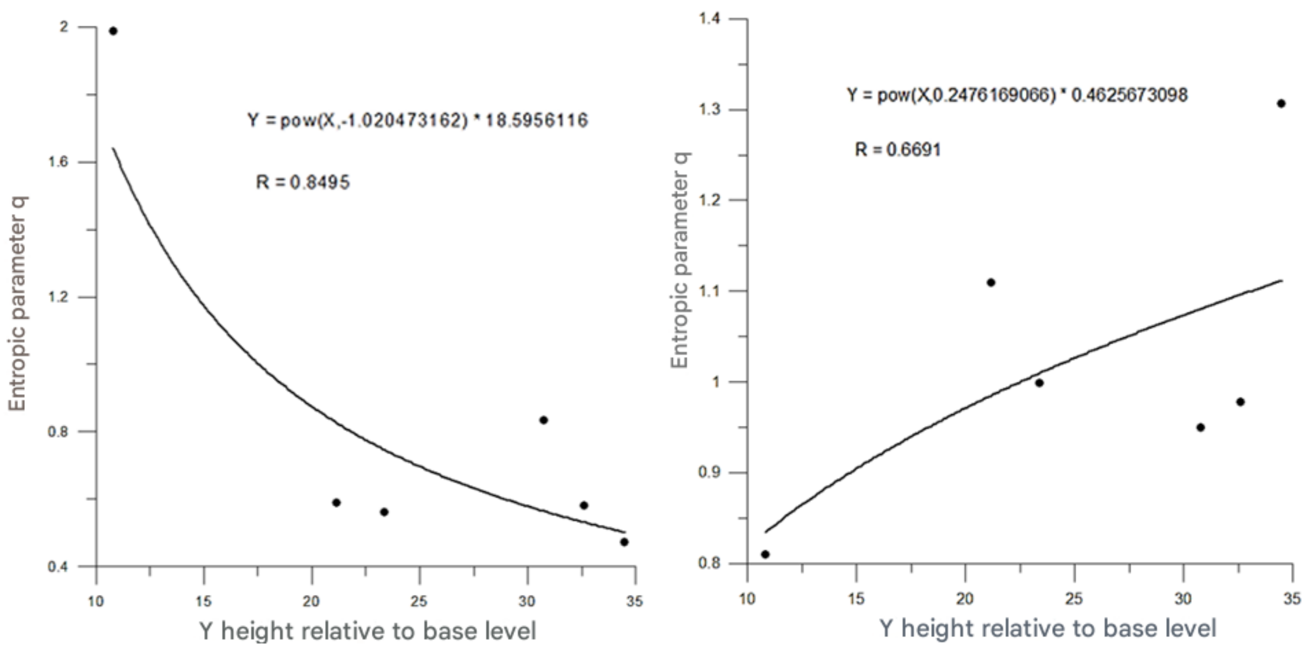

Figure 9 Variation of the entropic parameter δ with the height Y in relation to the base level

It is observed that the parameter q shows a tendency to decrease with the increase in the height of point Y, more prominent in the convex part, while parameter δ shows an opposite trend, that is, to increase with the increase of Y.

The graphs also suggest that the Gaussian statistic would be valid for different values of the height Y, whether it is q or δ. Thus, for the first, Y should be between 17 and 20 m, while for the second, between 20 and 25 m, because in these regions q and δ would be with values close to unity.

In addition, the graphs markedly suggest the validity of non-Gaussian statistics in the phenomenon studied, with all the implications arising from this finding (e.g., the cessation of the validity of the classical Central Limit Theorem (TSALLIS and UMAROV, 2022; TSALLIS, 2023).

COMPARISON OF THE RESULTS OF THE BASELINE SCENARIO WITH THOSE OF IGNÁCIO'S C2 PROFILE (2019)

The comparison of the results of case C2 of Ignácio's research (2019), with the results obtained in the Base Scenario of this research, are shown in Table 4.

Table 4 Comparison of Case C2 and Base Scenario

Entropy	Parameters	Values presented by Ignácio (2019)	Base scenario
Extended Q-Exponential	a'	0,1041	0,0547043
	b'	10,9006	10,799346
	C	54,6392	54,624439
	q	0,9803	0,8352076
	δ	0,8826	0,9501157
	Correlation coefficient	0,9982	0,9976617
Gaussian	The	0,1073	0,0563514
	B	10,6303	10,076701
	C	54,6382	54,632173
	Correlation coefficient	0,9965	0,9961701

The observation of Table 4 suggests that, despite small variations, the values are very close.

This small variation can be explained mainly by the fact of the considerations made in the modeling, such as the number of steps per rock, named in the software as *maximum steps per rock*. Ignacio in his thesis used 10,000 steps, while in this one 20,000 steps were used.

In addition, the software version used in this research is more recent, that is, the results may point to small differences due to possible fixes, new functionalities, features and performance. However, both sets of values suggest weakly chaotic behaviors.

CONDITIONS FOR THE VALIDITY OF NON-GAUSSIAN STATISTICS IN THE PROBLEM OF FALLING BLOCKS

As previously pointed out, statistics in power law are observed in physical phenomena characterized by long-range spatial and temporal memory, with the presence of some type of energy dissipation present in the phenomenon in question.

The results presented above fully corroborate these statements. More specifically, extended q-exponential distributions were observed in situations in which the blocks remained in contact with the rocky slope for a long time, in a permanent exchange of information.

In addition, the total energy that governs the phenomenon (most kinetic potential) proved to be insufficient, in the region located above point Y, to throw the unstable blocks away from the rocky slope, keeping them in contact with the rocky wall throughout the movement.

This suggests the possibility of the occurrence of partially elastic shocks of the unstable blocks with the slope, dictated by the normal and tangential restitution coefficients lower than unity, as shown in Araújo (2021). This leads to the damping of the system's available energy, and reduces the effects of inertia on the movement of the blocks, keeping them as if connected to the slope most of the time.

Figures 3, 4 and 5, relating, respectively, to Base Scenarios 1 and 2, suggest that the extended q-exponential distribution fits well to the experimental points, especially in its tail. In these cases, the total inadequacy of the Gaussian distribution is patent.

CONDITIONS FOR THE VALIDITY OF GAUSSIAN STATISTICS IN THE PROBLEM OF FALLING BLOCKS

On the other hand, as already pointed out, Gaussian statistics are observed in physical phenomena characterized by short-range spatial and temporal memory – or total absence thereof.

Specifically, in the phenomenon of falling blocks, this translates into the short contact time of the falling blocks with the rocky slope, and there is, therefore, little exchange of information between the blocks and the slope.

In addition, the total energy that governs the phenomenon (potential plus kinetic) is sufficient to throw the unstable blocks away from the rocky slope, the Y point constituting a trampoline.

This indicates the action of inertia effects of great magnitude, with approximately elastic and sporadic shocks between the blocks and the rocky wall (Araújo, 2021).

CONDITIONS OF VALIDITY OF STATISTICS RESULTING FROM THE LINEAR COMBINATION OF EXTENDED GAUSSIAN AND Q-EXPONENTIAL DISTRIBUTIONS IN THE BLOCK DROP PROBLEM

This time, the fall phenomenon is governed by the joint effects described in the two previous items, both by the dynamics upstream (above) of launch point Y (the aforementioned "trampoline effect"), and by the downstream dynamics of this same point. In more detail, in situations where the upstream dynamic keeps the blocks in contact with the slope (non-Gaussian statistics conditions), and the downstream dynamics throw them away from the rocky wall (Gaussian statistics conditions), the downslope result translates as a combination of these joint effects.

Figures 10, 11 and 12, related to Scenarios 3, 4 and 5, respectively, illustrate this perspective well, in which the combined distribution is shown in black, for comparison with the Gaussian and extended q-exponential distributions, all on a semi-log scale.

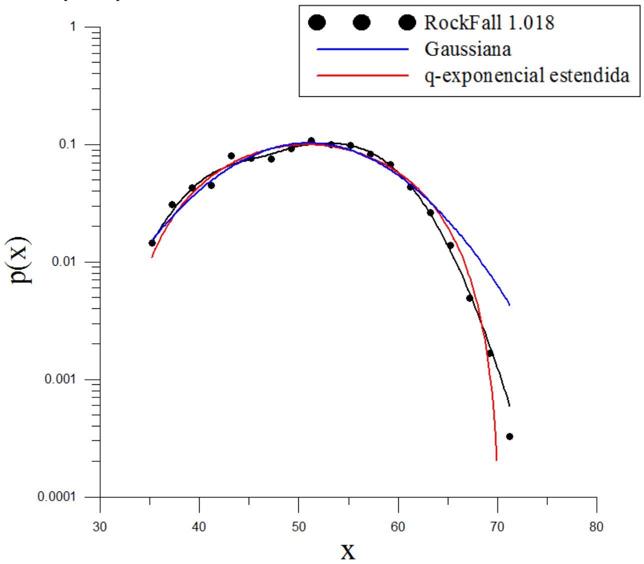

Figure 10 Log(p(x)) versus x Graph - Scenario 03

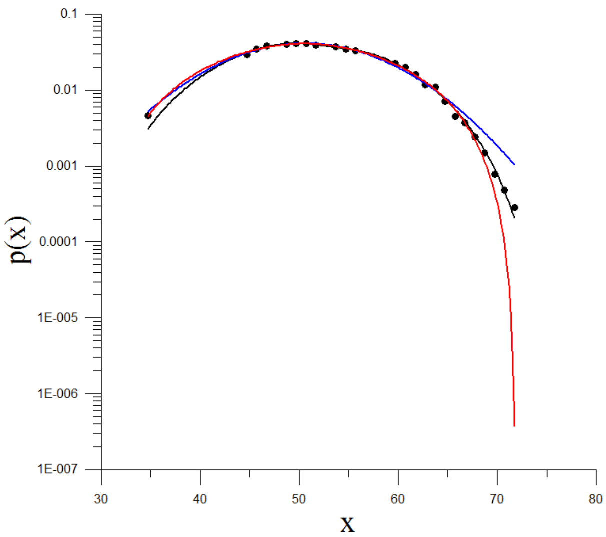

Figure 11 Log(p(x)) versus x Graph - Scenario 04

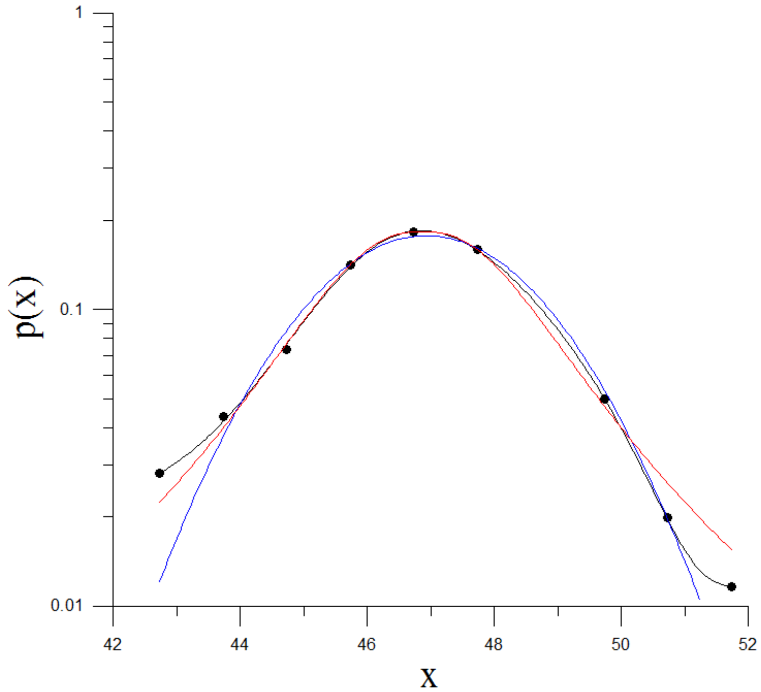

Figure 12 Graph log(p(x)) versus x - Scenario 05

The previous figures suggest that the combined distribution more faithfully portrays the experimental data from Scenarios 3, 4 and 5, which is why it is believed that both Gaussian and extended q-exponential distributions are acting together in the phenomenon of falling blocks.

The adjustment parameters for each case are presented in Table 5.

Table 5 Values found resulting from the sum of the distributions

Parameters	Scenario 3	Scenario 4	Scenario 5
a'	4,9872	0,0457	0,4707
b'	7,4325	11,0690	4,5152
C	49,8414	51,0351	49,8603
q	1,0067	0,7659	2,8460
δ	1,0025	1,1171	0,3414
the	-4,8910	-0,0062	-0,5475
b	-7,3655	4,0094	5,8351
c	49,8191	52,5424	51,4294
Correlation coefficient - R	0,9905	0,9991	0,9996

As can be seen, the adjustments are much more precise, especially in the region of the tails of the distributions, portrayed by the higher correlation coefficients.

It should be noted that the results of the analysis of the experimental data carried out in this research strongly suggest that the relative position of the concave and convex regions, which are part of a given geometry of a natural slope, has a marked importance in the response of the blocks down the slope and their respective statistics. That is, the order in which they are presented in the constitution of a given geometry of a natural slope has an influence on the statistics that govern the phenomenon.

CONCLUSIONS

The main conclusions reached at the end of this research are the following:

- The statistical analysis of the phenomenon of falling blocks is an important tool for understanding this intricate phenomenon, since it makes it possible to describe certain regularities and reiterations, which a deterministic analysis would not be able to perform, especially in a phenomenon dependent on so many parameters;
- In addition, the statistical analysis made it possible to clarify the validity regimes of Gaussian and non-Gaussian statistics. Specifically, non-Gaussian statistics occur frequently in long-range spatial and temporal memory phenomena, in which the unstable rock blocks remain in contact with the slope, effecting a continuous exchange of information and having the possibility of dissipating the total energy responsible for their dynamics;
- On the other hand, Gaussian statistics are characterized by describing physical systems governed by short-range spatial and temporal memory, in which the blocks spend very little time in contact with the rocky slope, considerably reducing the possibility of information exchange between the blocks and the rocky wall;
- This research also addressed the performance of the statistical distribution resulting from the linear combination of the Gaussian and extended q-exponential

distributions, proving to be more appropriate to describe rare phenomena, located at the tail of the distributions;

- The difficulties involving simulations in the RockFall2 Program were also pointed out, particularly when a very large number of block launches is required, aiming at a more robust statistics of experimental data. In this sense, it is perceived that this program was designed for deterministic analyses aimed at the design of containment structures (impact) dictated by good engineering practice;
- The situations analyzed in the different scenarios made it possible to evaluate the good performance of the non-Gaussian statistics, as well as the Gaussian statistics, the latter especially in the region of the warhead of the distributions. In particular, Base Scenarios 1 and 2 allowed us to observe the good performance of the extended q-exponential distributions, which configures destabilization phenomena classified as weakly chaotic (or complex);
- The situations analyzed in Scenarios 3, 4 and 5 also suggested responses in agreement with statistics of complex systems, except in the region of their tails, characterized by rare events, in which the combined distribution proved to be more efficient;
- The variation of the entropic parameters q and δ suggests that the first tends to have its values reduced when the height of the most prominent point of the convex region Y tends to increase, while the opposite happens with that second parameter, tending to have its values increased when the height of the point Y increases;
- Finally, it is worth mentioning that the results of the analysis of the experimental data carried out in this research strongly suggest that the relative position of the concave and convex regions, which are part of a given geometry of a natural slope, has a marked importance in the response of the blocks down the slope and in their respective statistics. Therefore, the order in which they are presented in the constitution of a given geometry of a natural slope has an influence on the statistics that govern the phenomenon.

SUGGESTIONS FOR FUTURE RESEARCH

Some themes for future research can be suggested, including:

- To significantly increase (in the order of 500,000 to 1,000,000) the number of entries in the statistical distributions used and to observe any discrepancies in the results, when compared to those already established by this line of research;

- Use of other statistical distributions (beta, e.g.) in the description of experimental data;
- Clarify in more detail the role of convex and concave profiles in the dynamics of unstable blocks;
- Investigate the divergence between initially very close launch trajectories and the evolution of their separation over time;
- Incorporate the influence of the fragmentation of the unstable blocks in the statistical response of the phenomenon;
- Incorporate the three-dimensional analysis of the phenomenon of falling blocks and investigate the emergence of any new behaviors not captured by the two-dimensional analysis.

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