MATHEMATICS APPROACHED FROM A COMPLEX PERSPECTIVE IN THE CLASSROOM

A MATEMÁTICA ABORDADA SOB UMA PERSPECTIVA COMPLEXA EM SALA DE AULA

LAS MATEMÁTICAS ABORDADAS DESDE UNA PERSPECTIVA COMPLEJA EN EL AULA

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ABSTRACT
The research described in this text was of a theoretical nature, based on Morinian complexity philosophy. The principle was adopted that languages are historical constructions carried out by both individuals and societies. Languages are not to be confused with the cognitive processes and products that correspond to them, but they go hand in hand with them. The hypothesis whose coherence we have sought to demonstrate in this scientific communication is that mathematics is a language and is developed, over time, with a view to achieving increasingly precise understandings, explanations and consensus on (among other things) phenomena and noumena. In other words: mathematics is human, and with it we try to move towards the understanding of so-called noumenic mathematics. The pedagogical contribution of this work consisted of the argumentative foundation of the proposal that, in mathematics classes, there should be reflections and debates about what knowledge is, particularly regarding what mathematical knowledge is, with emphasis, in such classes, on ideas supported by the philosophical system of the complexity.

Keywords: Complexity. Construction versus Discovery. Languages. Mathematics. Classroom.

RESUMO
A investigação descrita neste texto foi de caráter teórico, com alicerce na filosofia da complexidade moriniana. Adotou-se o princípio de que as línguas são construções históricas, a cargo tanto de indivíduos quanto de sociedades. Não se confundindo com os processos e os produtos cognitivos que lhes correspondem, as línguas, contudo, caminham pari passu com eles. A hipótese cuja coerência procurou-se mostrar nesta comunicação científica é a de que a matemática é uma língua e é elaborada, ao longo do tempo, com vistas à consecução de entendimentos, de explicações e de consensos, cada vez mais acurados, sobre (entre outras coisas) fenômenos e númenos. Ou seja: a matemática é humana, e com ela tenta-se rumar para a apreensão da matemática dita numênica. A contribuição pedagógica deste trabalho constituiu-se na fundamentação argumentativa da proposta de que, em aulas de matemática, haja reflexões e debates acerca do que é o conhecimento, particularmente a propósito do que é o conhecimento matemático, com ênfase, em tais aulas, a ideias apoiadas no sistema filosófico da complexidade.

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Palavras-chave: Complexity. Construction versus Discovery. Languages. Classroom. Mathematics.

RESUMEN

La investigación descrita en este texto fue de carácter teórico, basada en la filosofía de la complejidad moriniana. Se adoptó el principio de que las lenguajes son construcciones históricas, responsabilidad tanto de los individuos como de las sociedades. Las lenguajes no deben confundirse con los procesos y productos cognitivos que les corresponden, pero van de la mano con ellos. La hipótesis cuya coherencia hemos tratado de demostrar en esta comunicación científica es que las matemáticas son un lenguaje y se desarrollan, a lo largo del tiempo, con vistas a lograr comprensiones, explicaciones y consensos, cada vez más precisos, sobre (entre otras cosas) los fenómenos y los noúmenos. En otras palabras: las matemáticas son humanas, y con ellos intentamos avanzar hacia la comprensión de las llamadas matemáticas nouménicas. El aporte pedagógico de este trabajo consistió en la fundamentación argumentativa de la propuesta de que en las clases de matemáticas se realicen reflexiones y debates acerca de qué es el conocimiento, particularmente respecto de qué es el conocimiento matemático, con énfasis, en dichas clases, en ideas sustentadas en el sistema filosófico de la complejidad.

Palabras clave: Complejidad. Construcción versus Descubrimiento. Lenguaje. Matemáticas. Aula.

1 INITIAL CONSIDERATIONS

Languages, such as Portuguese and English, do not encompass the complexity and singularities of the natural/social world, the universe and/or *the noumena*[2]. Nevertheless, we consider them (in this case, we consider languages) as something plausible, notably in practical terms, insofar as they, for better or worse, are the resources we have for a series of daily and investigative purposes.

They can be and, in fact, are used, despite their limitations, to represent objects of interest and research, seeking (and achieving, in a certain way), through them, understandings, explanations and consensuses.

They are also used in a less pragmatic way, or even not at all pragmatic, when one aims to contextualize them in themselves. This is what we see in Portuguese, English, etc., combined with peculiar types of poetry and prose. Such specific and non-empirical modalities of poetic or prosaic Portuguese, as well as poetic or prosaic English, to stay only in these two languages, are not unscathed, however, to attempts and relative reaches of understandings, explanations and consensus.

Furthermore, there are no irremediable impediments with regard to subsequent and utilitarian applications of language excerpts consistent with the aforementioned modalities, or rather, there are no insurmountable obstacles regarding the applicability, conscious or not, of language extracts that were originally contextualized in themselves.

We agree with Abbagnano (2000), for whom languages are, on the one hand, systems or *structures* and, on the other, they suppose a *speaking mass* that transforms them into *social realities*.

Let us remember that a language is the result of constructions (Levy, 2022) over time, (constructions) from which – according to Levy (2019) – the individual or personal sphere (see the element or the *part*) and the collective or social sphere (see the whole or the *whole*) are not exempt.

We better accept the assertions that incorporate *part* and *all* by admitting, for example, that the individual (the *part*) is located in the heart of society and that, simultaneously, society (the *whole*), with its customs, with its rules, with its injunctions, with its myths and with its taboos, is within each individual [hologrammatic complex principle (Morin, 1999)].

[2] *Noumenon*: "In Kant's philosophy, a term that designates reality considered in itself – the thing-in-itself (*Ding-an-sich*), independently of the relation of knowledge, and can only be thought, without being known. It is opposed to phenomenon, which designates the sensible object precisely as the object of experience. The noumenon is thus the external cause of the possibility of knowledge, although it is, as such, by definition, unknowable" (Japiassú; Marcondes, 1996, p. 198).

Furthermore, we assume that the individual generates society, which, through the repercussion brought about by his attributes or qualities, generates the civility and/or humanity of the individual, in a *perpetuum mobile* [complex recursive principle (Morin, 1999)].

Finally, we are supporters of the notion that the individual and society are presented as *opposite sides of a coin*, and there is, therefore, antagonism and complementarity between them [complex dialogical principle (Morin, 1999)].

We acquiesce with the idea, hegemonic in the scientific community, that generic cognitive processes and products – including dynamics related to induction and deduction (Levy, 2016) – do not dispense with *historical languages* (that is, they do not dispense with languages) as representative structures (Abbagnano, 2000).

On the other hand, there is no doubt (we do not) that cognitive processes and products that denote specific techniques (e.g., mathematics, physics, etc.) depend analogously on representative supports, called *artificial languages* by Abbagnano (2000).

In the following pages, we intend to strengthen the hypothesis that mathematics is elaborated over time for (among other things) the achievement of understandings, explanations and consensuses that are increasingly closer to phenomena and *noumena*.

At the pedagogical level, the inquiry recorded in this article (which is of a theoretical nature) results in the proposition of mathematics classes interspersed with debates about mathematical knowledge, especially in the light of the philosophical theory of complexity.

2 HUMAN MATHEMATICS X NUMERICAL MATHEMATICS: CONSTRUCTION X DISCOVERY

There is *a distinction* between language and thought, but both, most of the time, walk inextricably *united* (Vygotsky, 1987) by the intricacies of *uncertainties* and singularities of the complexity of the world, (complexity) with which we are faced and/or which appeals to our attention. By the way, the triad *of distinction-union-uncertainty* is fundamental in the epistemological theory of complexity (Morin, 2001), (theory) that underpins this article.

Going a little further, we will adopt, in the present scientific communication, from this point on, the assumption that, when we explain *historical languages* (languages), we will also be discussing, *pari passu*, the thoughts that are based on them, even if we do not cite them openly.

In the same way, when we explain artificial *languages*, we will also be discussing, at the same pace or pace, the thoughts (see mathematical, scientific ideas, etc.) that are based on these languages, even if we do not make explicit reference to them.

To put it simply: from here on, what is valid for linguistic systems will be similarly valid for the processes and cognitive products allied to such systems; and vice versa: what concerns acts and cognitive effects will concern analogously the linguistic systems correlated to them.

In terms of mathematics studied in the basic and higher education cycles, we have applied (mathematics) (linked to the search for understandings, explanations and consensus about[3] phenomena of nature, the world around us and/or something that is of interest to us) and we have the so-called *mathematics for mathematics* or the so-called pure mathematics (which is still marked by the desire to achieve understandings, explanations and consensus).

The hologrammatic, recursive, and dialogical complex principles are, in our view, extendable to mathematics, which, like the creation of *historical languages* (see languages), results from both individual and collective efforts (Levy, 2019).

All in all:

a) there are not only applied *historical languages* (such as Portuguese, English, etc.), which are directed towards understandings, explanations and consensus of a pragmatic nature; but, equally, (in the case of Portuguese, English, etc.), there are *languages for languages' sake*, that is, there are elaborations, belonging to *historical* languages, which border on art or which, not infrequently, are themselves artistic, and this does not rule out pretensions aimed at understandings, explanations and consensus, as well as (this does not despise) dispositions for their use (or rather, dispositions, premeditated or not, for the use of these elaborations) in environments extrinsic to the artistic, in addition to (as we observe in *applied historical languages*[4]) not dispensing with individual and collective contexts for its formulation;

b) processes and products equal to those mentioned in item "a" take place in (and define the) mathematical scope.

As a simplifier, reductionist, fragmenter, determinist and/or standardizer par excellence, mathematics is still – and, perhaps, will be permanently – distant (despite its attempt, in the past and today, to get closer) from the complexity and singularity of phenomena, which are linked, from our point of view, to the complexity and singularity of *things in themselves* or of *numerical* reality (Kant, 2002), remaining (mathematics, which is,

[3] We accept the idea that phenomena depend jointly on *extra-human* or *noumenal reality* and on multiple human dimensions (biological-cerebral-perceptive, cognitive, psychological, sociocultural, etc.).
[4] From now on, in this article, we will refer to *historical languages* (and the cognitive mobilizations that are based on them) by using only the word *languages*; and we will mention *artificial languages* (together with the cognitive activities that are based on them) by means of grammatical particles that make them explicit, such as mathematics, philosophy, sciences, etc.

as we advocate, engineered by man), instead attuned to an unreal, ideal or idealized, regulated and simplified world.

We suppose that mathematics, for the reasons listed in the lines above (i.e.: simplification, reductionism, fragmentation, determinism and/or standardization), is also far from *the numerical reality*, since we defend, in accordance with Kant (2002), the principle that there is a certain portion of a link and/or that there is a relationship, to a certain degree, between phenomenon and *nouenon*.

The areas of knowledge that are based on mathematics have their dialogues with phenomena and, consequently, with reality (Note: reality is always beyond phenomena), restricted or hindered by the reasons listed above.

Agreeing with the feasibility of having a portion (despite knowing nothing or almost nothing about the magnitude of this portion) of connection between phenomena and *numbers* (Kant, 2002), we do not shy away from the idea that we create knowledge (including mathematical knowledge) so that, among other things, we can better understand the phenomena and, in the course of time or history, we can discover or try to discover, more and more, aspects of reality, which transcend phenomena (Levy, 2022).

We are thus faced with a contradiction or an antagonism: historically, *we construct* interpretations or representations aimed at discovering supposedly absolute truths.

The philosophical system of Morinian complexity admits, supported by the dialogical principle (made explicit, in previous lines, through the duo composed of individual and society, when we discussed languages), occurrences that are at the same time antagonistic and complementary. In these terms, creations and discoveries oppose and, at the same time, complement each other.

Further adjusting the creation-discovery dyad to the philosophy of complexity, advocated by Edgar Morin, it is up to us to state that:

a) creations entail discoveries, which, in turn, act back on creations, (re)generating them or contributing to their (re)generation, in an uninterrupted circular path (complex recursive principle), as observed in mathematical elaborations, which, among several of their functions, lead us and/or aspire to lead us to the discovery of *numerical* reality; This reality (to be) discovered is, in our judgment, the generator of man and his creative efforts (among them, mathematical creative efforts);

b) creations are in discoveries, just as discoveries are in creations (hologrammatic complex principle), such as mathematical processes and products, which are part of man (Note: human beings, individually and collectively, do not fail to encompass their own constructions and/or do not fail to encompass them), which, in turn, we believe belong to *the noumenal* reality to be discovered in its essence (Note: man, both individually and collectively, is an integral part of the world, of nature, of the universe); And this reality (to be discovered in its details), with its attributes or with its properties, is found, in our view, within man, in particular, and society, as a whole, and is therefore located in the intimacy or in the core of the creations of humanity, insofar as creator (man/society) and creation (mathematics, science, etc.) identify each other.

Given our appeal to the dialogue between creation and discovery, it seems reasonable to us, then, the idea that, in the course of time or history, through increasingly well-elaborated constructions, we approach and/or seek a continuous approximation of reality. This would be true for languages, mathematics and science.

In terms of a notorious portion of its attributions, mathematics[5], in its applied form, would refer to a set of constructions (processes and products) with which we aim, along the temporal or historical path, to discover or close to the discovery of *numerical mathematics*.

The latter would not be denotative, it is good to emphasize it again, of current mathematics (pure or applied) and its respective simplifications, reductions, fragmentations, certainties and/or standardizations, since we conceive *the numenic reality* (which would include *numenic mathematics*) as something complex to the extreme, which is signaled to us by the very high complexity of the phenomena with which we are confronted, which are dynamics pertaining, to a certain extent, we believe (as we have already mentioned in this text), to this *numerical reality*.

3 SUGGESTIONS FOR THE CLASSROOM

Languages are simplifying, reductionist, fragmenting, deterministic and/or standardizing, not entirely contemplating the singularity and complexity of natural phenomena and/or events; phenomena/events that, in some way, they propose (and such a proposition is not their only function) to represent and/or interpret.

The limited potential of languages (although there has always been and, apparently, there will continue to be efforts to evolve their effectiveness) and the timid reach, with them, of the phenomena that surround us, and/or of the events that call our attention, are topics of little or no interest in discussions or debates in schools and university mathematics courses.

The abyss separating what is desired and what is achieved is, it seems, enormous – although the human being does not have, and possibly never will have, an indisputable notion of the dimension of the aforementioned enormity – and it is urgent that students at the various

[5] In this article, the grammatical mathematical particle, when it is not specified or adjectivated, should be understood as *human mathematics* or *man-made mathematics*, as opposed to *numerical mathematics*.

levels of education become aware of the linguistic, interpretative or representative limitations that contribute to this separation.

In any case, languages constitute an acceptable undertaking, because (we ratify that) they achieve effects evaluated as relatively satisfactory. Such satisfaction is indicated by official or unofficial agreements congruent with the public's eagerness involved in the use of the linguistic-cognitive structure in focus. With languages, we aim to understand, explain and reach consensus. Despite the incompleteness that marks communication in general (and idiomatic communication in particular), we know that understanding, explanation and consensus are obtained, at a given level, with the support of languages.

We reiterate that such discussions about the incompleteness of communication often go beyond what is debated in the classroom, especially outside the meetings or educational meetings in which the faculty and students deal or work with mathematics.

Bearing in mind that languages and mathematics – which is a specific technique pertinent to an *artificial language* (Abbagnano, 2000) – are linked to certain concepts, such as incompleteness and consensus/agreement, is essential for the apprehension, in school and in academia, of both meanings: that of languages and that of mathematics.

Languages have a pragmatic purpose (let's call them, in this case, applied languages) and, let's say, an artistic purpose (let's call them, here, *languages for languages' sake*, or rather, *languages applied to themselves*), and extracts from *languages applied to them* (as seen in certain fragments of prosaic Portuguese or poetic Portuguese, depending on what we have described in earlier parts of this article), After being created, they are sometimes contextualizable, through uses and customs (consciously established or not), in lands different from those originally planned for them.

One could, in the classroom, draw a parallel of correspondence between such feasibility of contextualization and that which is suggested and executed in / with mathematics (when, deliberately, but also, not infrequently, contingently – thanks to some *happy accidents* – one migrates from the pure to the applied domain). Reflections on this parallel, opening the way to evidence the mathematical construction, have, we think, the capacity to inspire successful arguments in the face of the misleading idea that mathematics is a discovery.

In both situations (applied languages and pure languages), there is the individual and the collective (that is: man, in particular, and society, as a whole) working to generate idiomatic processes and products. We consider, therefore, that languages result from human constructions or elaborations in which the singular and plural categories are participants, in creative flows that are at the same time unidimensional and syncretic, a fact that lacks more detailed mentions and debates in school and university.

Part and whole (individual and society, for example) are subject to consideration, in the classroom, in tune with the complex dialogical, hologrammatic, and recursive principles.

Processes and products identical to those that make up languages characterize, from our point of view, mathematics:

a) there is applied mathematics, which, analogous to applied languages, is intended to interpret or represent phenomena related to everyday life, the world and/or nature;

b) a priori turned to themselves, *languages for languages' sake* can be further contextualized in a partially operative way in other spheres. In the same way, pure mathematics (*mathematics for mathematics*'s sake), which does not initially have practical objectives, adds chances of causing remarkable applications.

Comparing the applications and abstractions of languages with the applications and abstractions of mathematics, emphasizing their possible and probable connections, is not a routine action in the school and university universes, which is regrettable, since a comparison of this kind would provide admirable epistemological acquisitions to students in the middle of mathematics class.

We insist/repeat that:

a) like languages, mathematics is the result of both individual and collective effort;

b) as with languages, the creation of mathematics takes place in the course of time or history.

With languages and mathematics (and also with the sciences and other *artificial languages*), we try, among other things, to approach a minimally obscured reading of phenomena, which mediate our relationship with *the noumena*, which, in turn, denote an *extra-human reality* or, we dare to consider, an ultimate truth, perhaps intangible by us.

In philosophy classes, usually without relevant appeal to mathematics, this hypothesis is often debated. In math classes, on the other hand, a conversation of this order tends not to be fostered. We are adept at teaching mathematics interspersed with epistemological comments. Not detracting from the importance of knowing, we reiterate the urgency of *knowing knowledge*, especially – given the part that concerns us – mathematical knowledge.

4 FINAL CONSIDERATIONS

Languages, sciences and mathematics, both pure and applied, are, on a large scale, constructed so that we can master or seek to master phenomena, aiming at understandings, explanations and consensuses that, in the last instance of progress, would be identified with reality *itself*.

We march from the construction of interpretations about phenomena [6]towards the discovery of something that is independent of the human spirit. *Knowing knowledge*, with emphasis on *mathematical knowledge*, should, for us, extrapolate the sphere of philosophy classes and philosophy classes of mathematics (education), becoming part of the syllabus or syllabus of mathematics classes.

The *teaching-student* exercise that brings together philosophy and mathematics in mathematics classes is, in our opinion, potentially leveraging unusual contributions to both philosophical and mathematical knowledge.

We emphasize that human mathematics, due to its simplification, its reductionism, its fragmentation, its determinism and/or its standardizations (as much as we have tried and succeeded, over time, to raise it in terms of complexity), is still far behind *numerical mathematics*. In this sense, we do not venture to affirm that the human and the *noumenal*, one day, will be able to dialogue without the intermediary of the phenomenal.

For us, *numerical mathematics* is or would be one of the components of *reality itself*. We imagine that making students aware of this philosophical position would lead them to questions and stimulate their learning of mathematics and philosophy during mathematics classes.

We postulate and ratify that we have created (see human mathematics) aiming to discover (see *numerical mathematics*). The dyad creation x discovery, like the dyad part/individual x whole/society, obeys the *complex*[7] dialogical, hologrammatic, and recursive principles.

If we admit that historical languages (languages) result from human constructions (Note: and it is common for us to admit it!), then it will be coherent, in view of all that we have exposed in these pages, to assume that the same happens with mathematics and science. We suppose that the defense of the aforementioned idea, before the students, is an instigating action, in them, of reflections on the nature of mathematics, science and/or artificial languages in general.

We understand that the teacher, especially the mathematics teacher (as it is the *core discipline* of this article), is able to address such epistemological elements in the classroom.

Obviously, complexity theory is an alternative approach to *knowledge about knowledge* with mathematics students, but it is not the only one. Its role in this theoretical work is central because it is the philosophical theory with which we most identify, with

[6] We reinforce that phenomena are not alien to our subjective interference, although they are not limited to it.
[7] Such complex principles also govern the pair formed by pure languages and applied languages, as well as the pair composed of pure mathematics and applied mathematics.

emphasis on our part, in the present scientific communication, to epistemological aspects concerning, of course, mathematics.

The pedagogical contribution announced in these pages was, we repeat, the argumentative foundation of the proposal that, in mathematics classes, there should be critical thoughts and/or debates about what knowledge is, in particular what mathematical knowledge is from the perspective of Morinian complexity.

Naturally, our idea, like any and all original suggestions, is likely to encounter resistance to its implementation, including and perhaps mainly, resistance from members of the faculty itself. But we are sure that any reluctance will not prevent us from publicizing such a path, certainly unprecedented with regard to the teaching and learning of mathematics. We are available to those who are interested in our idea and we will always be open to collaborations aimed at improving it.

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