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MATHEMATICAL MODELING AND SCENARIOS FOR INVESTIGATION: THE DEVELOPMENT OF A CRITICAL STANCE TOWARDS THE SOCIAL REALITIES OF STUDENTS

MODELAGEM MATEMÁTICA E CENÁRIOS PARA INVESTIGAÇÃO: O DESENVOLVIMENTO DA POSTURA CRÍTICA FRENTE ÀS REALIDADES SOCIAIS DOS ESTUDANTES

MODELIZACIÓN MATEMÁTICA Y ESCENARIOS DE INVESTIGACIÓN: EL DESARROLLO DE UNA POSTURA CRÍTICA ANTE LAS REALIDADES SOCIALES DE LOS ESTUDIANTES

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ABSTRACT

This paper aims to present the contribution of Mathematical Modeling to the construction of different Scenarios for Investigation in the teaching and learning of Mathematics in higher education courses. The search for different Scenarios for Investigation can stimulate student engagement through activities that encourage their connection with real-life situations and their professional experiences. Thus, we highlight the guiding question of this research: What are the contributions of Mathematical Modeling in the construction of different Scenarios for Investigation in higher education? According to our studies, we found different scenarios constructed according to different mathematical entities and with a high degree of reference to real life. It is worth noting that, in all the situations studied, we considered the interdisciplinary aspect between mathematical objects, undergraduate programs, and, above all, how this aspect presents itself in relation to the possible professional roles of future graduates. To support the definition of different Investigation Scenarios, we base ourselves on the theoretical assumptions of Skovsmose (2000) and Alro and Skovsmose (2010), in addition to the main ideas adopted by Bassanezi (2010) and Biembengut and Hein (2000) regarding teaching and learning with Mathematical Modeling. To this end, we rely on academic works selected from CAPES databases that considered the teaching and learning of mathematical objects in relation to the construction and analysis of models. In this sense, we adopted an investigative stance based on documentary research in a qualitative approach. Among the research analyzed, we highlight Investigation Scenario 1, portrayed in this work, which evidences the construction of a Mathematical Model for studying the lifespan of an overpass as a function of local vehicle flow. The constructed model pointed to mathematical arguments that support the group's estimates, and its analysis was supported by the students' critical stances on the well-being of the population and the use of public funds. We consider that mathematical models encourage learning based on different Scenarios for Investigation, such as those related to social functions or other problems existing outside the educational universe that are close to the students' experiences.

Keywords: Mathematical Modeling. Scenarios for Investigation. Higher Education.

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RESUMO
O presente tem a proposta de apresentar a contribuição da Modelagem Matemática frente a construção de diferentes Cenários para Investigação no ensino e na aprendizagem da Matemática em cursos superiores. A busca pelos diferentes Cenários para Investigação poderá estimular o envolvimento do aluno a partir de atividades que instiguem sua aproximação com situações reais e de suas vivências profissionais. Assim, destacamos a questão norteadora desta pesquisa: Quais as contribuições da Modelagem Matemática na construção de diferentes Cenários para Investigação no ensino superior? De acordo com nossos estudos, encontramos diferentes cenários construídos segundo diferentes entidades matemáticas e com alto grau de referência a vida real. Vale destacar que, em todas as situações estudadas, consideramos o aspecto interdisciplinar entre objetos matemáticos, às graduações e, sobretudo, como este aspecto se apresenta frente às possíveis atuações profissionais do futuro egresso. Para que possamos subsidiar a definição de diferentes Cenários para Investigação, fundamentamo-nos nos pressupostos teóricos de Skovsmose (2000) e Alro e Skovsmose (2010), além disso, acrescentamos as principais ideias adotadas por Bassanezi (2010) e Biembengut e Hein (2000) a respeito do ensino e da aprendizagem com Modelagem Matemática. Para tanto, apoiamo-nos em trabalhos acadêmicos selecionados em bases de dados da CAPES que considerou o ensino e aprendizagem de objetos matemáticos frente à construção e análise de modelos. Nesse sentido, adotamos uma postura investigativa com base em pesquisas documentais numa abordagem qualitativa. Dentre as pesquisas analisadas, destacamos o Cenário para Investigação 1 retratado neste trabalho que evidencia a construção de um Modelo Matemático para estudo de tempo de vida de um viaduto em função do fluxo de veículos local. O modelo construído apontou para argumentos matemáticos que sustentam as estimativas do grupo e sua análise foi suportada por posturas críticas dos alunos sobre o bem-estar da população e sobre o uso do dinheiro público. Consideramos que os modelos matemáticos incentivam a aprendizagem com base em diferentes Cenários para Investigação como os relacionados às funções sociais ou demais problemas existentes fora do universo educacional que estejam próximos das experiências dos estudantes.

Palavras-chave: Modelagem Matemática. Cenários para Investigação. Ensino Superior.

RESUMEN
Este artículo tiene como objetivo presentar la contribución de la Modelización Matemática a la construcción de diferentes Escenarios de Investigación en la enseñanza y el aprendizaje de las Matemáticas en cursos de educación superior. La búsqueda de diferentes Escenarios de Investigación puede estimular la participación estudiantil a través de actividades que fomentan su conexión con situaciones de la vida real y sus experiencias profesionales. Por lo tanto, destacamos la pregunta guía de esta investigación: ¿Cuáles son las contribuciones de la Modelización Matemática en la construcción de diferentes Escenarios de Investigación en la educación superior? Según nuestros estudios, encontramos diferentes escenarios construidos según diferentes entidades matemáticas y con un alto grado de referencia a la vida real. Cabe destacar que, en todas las situaciones estudiadas, consideramos el aspecto interdisciplinario entre los objetos matemáticos, los programas de pregrado y, sobre todo, cómo este aspecto se presenta en relación con los posibles roles profesionales de los futuros graduados. Para fundamentar la definición de los diferentes Escenarios de Investigación, nos basamos en los supuestos teóricos de Skovsmose (2000) y Alro y Skovsmose (2010), además de las ideas principales adoptadas por Bassanezi (2010) y Biembengut y Hein (2000) respecto a la enseñanza y el aprendizaje con Modelado Matemático. Para ello, nos apoyamos en trabajos académicos seleccionados de las bases de datos de CAPES que consideraron la enseñanza y el aprendizaje de objetos matemáticos en relación con la construcción y el análisis de modelos. En este sentido, adoptamos una postura investigativa basada en la investigación documental con un enfoque cualitativo. Entre las investigaciones

analizadas, destacamos el Escenario de Investigación 1, presentado en este trabajo, que evidencia la construcción de un Modelo Matemático para estudiar la vida útil de un paso elevado en función del flujo vehicular local. El modelo construido apuntó a argumentos matemáticos que respaldan las estimaciones del grupo, y su análisis se sustentó en las posturas críticas de los estudiantes sobre el bienestar de la población y el uso de fondos públicos. Consideramos que los modelos matemáticos fomentan el aprendizaje basado en diferentes escenarios de investigación, como los relacionados con las funciones sociales u otros problemas ajenos al ámbito educativo que son cercanos a las experiencias de los estudiantes.

Palabras clave: Modelado Matemático. Escenarios de Investigación. Educación Superior.

1 INTRODUCTION

The purpose of this research is to present the contribution of Mathematical Modeling in the construction of different Scenarios for Investigation in the teaching and learning of Mathematics in higher education courses. We consider that understanding Mathematical Modeling guided by a perspective of Critical Mathematics Education can reorganize environments in which students use mathematics to solve problems originating in reality. This approach allows the solution found to be problematized and questioned and to be based on reflections, the identification of the variables included, and the real reasons for their choices.

For Skovsmose (2010), radical changes that have occurred in the classroom have allowed the traditional methodology employed to be replaced by thematic approaches and work with projects, referring to the idea of Critical Mathematics Education. According to the author, "Critical Mathematics Education is concerned with the way in which Mathematics in general influences our cultural, technological and political environment and with the purposes for which mathematical competence should serve" (Skovsmose, 2010, p. 18).

Skovsmose (2010) consolidates the idea that there is a need to adopt a critical stance to be assumed by Mathematics Education and that it should not be based solely on textbooks. He also highlights that critical thinking through mathematics becomes relevant, since it is always present and everywhere.

The aforementioned author argues that from Mathematics in action it is possible to establish hypothetical situations, study their particular details, in addition to grounding the constitution of a wide variety of social phenomena that make Mathematics part of society.

Thus, in traditional models of mathematics teaching, the communication patterns between teacher and student become repetitive and, some, dominant. Therefore, it is essential to break such communication patterns based on investigative approaches that discuss activities as action.

In this sense, our research question is: What is the contribution of Mathematical Modeling in the face of the construction of different Scenarios for Investigation?

With the expectation of providing new forms of learning from the active and effective participation of the student in the construction of their knowledge, we evidence the identification of these scenarios built from mathematical models that instigate more complex, unpredictable investigation processes that aim at the construction of knowledge and communication practice.

2 RATIONALE

Unlike the traditional approach based on the solution of exercises in class, on the role of the teacher as disseminator of the content and on the role of the students as passive receivers of this information, *problem solving, thematic approaches, work with projects, etc*. (ALRO and SKOVSMOSE, 2010, p.52) have instigated researchers in the search for investigative approaches that discuss the activity no longer as an inspection agent.

Among the investigative approaches, we consider that the mathematical models built from the perspective of a Critical Mathematics Education will allow to congregate and involve students and teachers around common ideas and objectives, making their participation active. In addition, the collection and capture of a high volume of well-crafted information can contribute to the development of critical thinking, creativity and initiative in the search for a solution to these models. It is worth mentioning that these models may constitute possible Scenarios for Investigation.

From the perspective of Skovsmose (2000) and Alro and Skovsmose (2010), Research Scenarios are those that invite students to formulate questions and seek explanations, allowing both students and teachers to engage in interdisciplinary environments inherent to their academic and professional backgrounds.

For Fazenda and Godoy (2023, p. 31),

> Interdisciplinarity is a category of action of reflective doing, of what happens between two or more people and objects. It is the subject-object relationship and between subjects through which it is possible to integrate and uniquely know from zones of permeability of dialogues and the reading of different worlds.

Interdisciplinarity requires, in practice, a deep immersion in daily work and the understanding of hidden aspects of the act of learning and the aspects expressed, putting them into question (FAZENDA and GODOY, 2023). In this sense, interdisciplinarity allows a change in behavior, inviting us to new experiences.

Also according to the authors, although official documents point to the relevance of interdisciplinary aspects related to the multidisciplinarity of social, economic and cultural factors, in practice, it is common to see students establish the relationships between the various contents and disciplines of the course on their own. In this way, in an attempt to meet the complexity of the situations experienced by individuals, school mathematics defends the critical participation of students in society and in their integral formation as citizens.

3 THEORETICAL FOUNDATION

As theoretical assumptions of our investigation, we consider the main ideas of Bassanezi (2010) and Biembengut and Hein (2000) regarding Mathematical Modeling and Skovsmose (2000) and Alro and Skovsmose (2010) regarding the investigative approach promoted by Scenarios for Investigation.

3.1 THE ROLE OF MATHEMATICAL MODELING

According to Bassanezi (2010), Mathematical Modeling can be considered both as a scientific research method and as a teaching strategy. When referring to Mathematical Modeling as a scientific method or research instrument, it is possible to consider that it can stimulate the construction of new ideas and experimental techniques. In addition, it is a method that allows extrapolations, interpolations and forecasts, in addition to suggesting priorities for resource applications and research, thus promoting better decision-making. When we refer to modeling as a teaching and learning strategy, we can consider the emphasis of undergraduate courses on developing applicable mathematical disciplines, and of stricto-sensu graduate courses that seek to bring the student's mathematical creativity in the study of phenomenological situations based on mathematical models.

Biembengut and Hein (2000) point out that the need to restructure curricula and teaching methods aims to develop the students' potential based on the ability to think critically and independently.

However, Bassanezi (2010) explains that a large part of mathematical knowledge has been built exclusively within the field of Mathematics without concern for the external use of its knowledge. The author adds that there is currently a consensus among different professionals who consider that the skills of a physicist, an engineer, or a biologist are allied to mathematical competence. In view of the above, it becomes possible to apply this pattern of thought to different areas of knowledge that start to consider that their theories are modeled through mathematical language.

A Mathematical Model is defined as "a consistent set of equations or mathematical structures, elaborated to correspond to some phenomenon – this can be physical, biological, social, psychological, conceptual or even another Mathematical Model" (BASSANEZI, 2010, p. 174)

The author considers that mathematical concepts and structures can be adapted to the phenomena of reality, just as the phenomena of reality can serve as a source of obtaining new concepts and mathematical structures. There is, therefore, a combination of both alternatives.

For Biembengut and Hein (2000), the formulation of models requires a detailed mathematical formulation consisting of a set of symbols and mathematical relations that translate a phenomenon or problem. Such models are described by numerical expressions, formulas, diagrams, graphs, geometric representations, tables, etc. However, it is worth noting that the models come from approximations that, even in a simplified view, seek to portray the reality of the researched situation. It is up to the teacher or researcher to identify which mathematical content best suits the proposed problem, based on their intuition, creativity and knowledge and, thus, adopt the best available strategy that will be submitted to each of the steps necessary for the construction of the model.

Biembengut and Hein (2000, p.13) cite Mathematical Modeling as "[...] an art, by formulating, solving and elaborating expressions that are valid not only for the particular solution, but that also serve, later, as a support for other applications and theories".

Mathematical Modeling is a dynamic process used to obtain and validate mathematical models that reflect a portion of reality from the selection of arguments and parameters considered essential.

Biembengut and Hein (2000) add that the procedures for constructing a model can be grouped into three well-defined stages according to Table 1:

Table 1

Stages of Mathematical Modeling

1. Interaction	**2. Mathematization**	**3. Mathematical Model**
1.1 Recognition of the problem situation	2.1 Formulation of the problem = > hypothesis	3.1 Model Interpretation
1.2 Familiarization: search for theoretical references that support the proposed problem.	2.1.1 Classify the information (relevant and non-relevant) and identify the facts involved;	3.2 Validation
	2.1.2 Decide which factors to be pursued based on the hypotheses;	
	2.1.3 Select the relevant and constant variables involved;	
	2.1.4 Select appropriate symbols for these variables;	

	2.1.5 Describe relationships in mathematical terms.	
	2.2 Problem solving in terms of the model	

Source: prepared by the author, 2024.

Table 1 presents the steps described by Biembengut and Hein (2000) for the construction of a Mathematical Model. The first stage, called Interaction, consists of the recognition of the problem-situation and the study of the theoretical framework that will support the construction and subsequent analysis of the data and information obtained. In the second stage, called Mathematization, there is the raising of hypotheses of the problem-situation, as well as the investigation of which variables, constants and relationships will be described in the model. It is at this stage that the construction and resolution of the Mathematical Model also occurs. And, finally, in the third stage, there is an analysis of the implications of the solution found in the proposed Mathematical Model and its validation from the identification of the degree of approximation and reliability with the problem-situation. It is important to note that, if the model does not meet the needs, it is important to return to the Mathematization stage and check the adjustment of hypotheses, variables, etc.

Bassanezi (2014) points out that an ideal model is one that not only explains the results but is also capable of predicting new results or situations that do not match the expected. In many cases, the model needs to be reformulated based on modifications in the variables or in the previously established laws of formation.

With the proposal to identify how Mathematical Modeling can contribute to the creation of different Research Scenarios, it is up to us to point out the main considerations about these scenarios from the perspective of Skovsmose (2000) and Alro and Skovsmose (2010).

3.2 RESEARCH SCENARIOS AS LEARNING ENVIRONMENTS

For Skovsmose (2000), the Scenario for Investigation instigates an environment that can support an investigative work based on the formulation of questions such as: "What happens if ...?", or "Why is this ...?" that can be instigated from the valorization of dialogue. Thus, the Scenario for Investigation constitutes a new learning environment, as it invites students to formulate questions and seek explanations, making them involved in the process of knowledge exploration.

They also point out that many studies on communication still focus on traditional Mathematics classes, being understood as traditional, the environment in which textbooks occupy a central role, where teachers bring new content and students solve exercises. The act of correcting and finding errors characterizes the general structure of the class.

However, it is noticeable that radical changes have taken place in the classroom, as there is a gradual replacement of the traditional methodology by thematic approaches and by work with projects that refer to the idea of Critical Mathematics Education. SKOVSMOSE (2000) "states that Critical Mathematics Education is concerned with the way in which Mathematics in general influences our cultural, technological and political environment and with the purposes for which mathematical competence should serve" (p.18).

In traditional models of mathematics teaching, the patterns of communication between teacher and student become repetitive and dominant. The exercise paradigm is a unique aspect of traditional Mathematics, playing a fundamental role in the organization of classes, in the pattern of communication between teacher and student and in the role that Mathematics plays in society as a whole. An example of this scenario is when Mathematics has a supervisory function based on exercises that, in general, are prepared by authors of textbooks, turning them into pre-established elements as a practice in the classroom. However, according to Skovsmose (2000), the exercise paradigm has "been challenged by problem solving, thematic approaches, work with projects, etc." (p. 52) that correspond to a set of methodologies expressed by the author as investigative approaches. The activity should be discussed as an action and not as a compulsory activity as exercises are.

Table 2 is presented below, which represents a simplified model of different forms of reference and learning environments:

Table 2

The different learning environments

	Exercise Paradigm	Scenarios for Investigation
References to pure mathematics	(1)	(2)
References to semi-reality	(3)	(4)
Real-world references	(5)	(6)

Source: Adapted from Skovsmose, 2000

For Alro and Skovsmose (2010) and Skovsmose (2000), the exercises that are situated in a semi-reality (3) express an artificial situation and are presented based on an agreement established between teacher and student. Among the principles present in this agreement is that the semi-reality is completely described in the statement of the exercise and no external information is relevant to its resolution. In this sense, semi-realities are worlds without sensory impressions and with exact values, since they are solely defined by measurements. There is no discussion of the context and the goal is to find a single correct answer.

The authors present that there are also exercises that refer to pure mathematics (1) and that are described by imperative formulations such as, for example, "Solve the question ...", "Reduce the expression ...", or even, "Build the figure ... “. They consider that all the activities mentioned are linked to the exercise paradigm and that "the teaching of traditional mathematics is closely associated with the resolution of exercises referring to pure mathematics or semi-realities" (Alro and Skovsmose, 2010, p. 55). There are also reality-based exercises that offer a type of learning (5).

They also emphasize that a great effort has been made when it is proposed to use real-life data in the elaboration of exercises, thus starting to ponder on the reliability of the calculations and on the information that the exercise presents. In this way, reality-based exercises open a gap in the traditional teaching of Mathematics because it allows the questioning of the information contained in them.

Leaving the exercise paradigm ((1), (3) and (5)), we move on to a very different learning environment that constitutes the Research Scenarios ((2), (4) and (6)). In them, students participate in the investigation process and formulate questions such as, for example, *What happens if ...?,* or even, *Why is it this way ...?* The Scenarios for Investigation can also be related to semi-realities (4), but this environment is no longer used as a resource for the production of exercises, but for students to move on to new explorations and explanations, enabling the construction and improvement of strategies by the students. Other scenarios point to real-life situations (6) that are found in the current that adopts work with projects in Mathematics Education.

Skovsmose (2000) adds that leaving the exercise paradigm and working based on different Research Scenarios makes teachers and students leave their comfort zone and enter a risk zone. And he poses the following question: "What are the possible gains of working in a risk area associated with a research scenario?" (p.58). As an answer, the following are pointed out: possibility of greater involvement and cooperation of students, different communication patterns and new learning qualities in an open process associated with the Scenario for Investigation.

However, it is worth mentioning that a good part of Mathematics Education still alternates environments (1) and (3), showing a bleak picture of what happens in the traditional classroom. The studies reveal that there is no recognition of the possibility of other learning environments allowing which allows a differentiation between the tradition of school mathematics and the tradition of investigative mathematics.

The author adds that the learning environment of uncle (6) has challenged the tradition of school mathematics, however, he encourages that the challenges also be organized in terms of the learning environments of types (2) and (4). It shows that the environment of type (6) is not the only alternative to the exercise paradigm. In fact, "I don't want to suggest that a learning environment represents the ultimate goal for critical mathematics education or not." (SKOVSMOSE, 2000, p.15).

Thus, it is pertinent to state that Mathematics Education should move between the different environments of the matrix and that the exercises should not be abandoned. The learning environments matrix can be used as an analytical instrument to identify which different learning environments have been experienced, what time is spent by one or two environments, and what difficulties are encountered in moving from one to the other.

There must be a harmony between the parameters of a learning environment, that is, between the way that "meaning is produced, tasks are organized, the textbook is structured, communication is developed, etc." (SKOVSMOSE, 2000, p. 17). In addition, the movement between different learning environments, especially towards Research Scenarios, will cause a high degree of uncertainty that should not be eliminated.

4 METHODOLOGY

In order to answer the guiding question of this research, we sought to describe data from a qualitative investigation.

Borba (2010) states that qualitative research in Mathematics Education has been a great challenge since Mathematics teachers work with quantities. However, attention should be paid to the type of information you want to obtain. In a qualitative investigation, researchers are more interested in the process and not only in the results or products. Relevance is given to the meaning that is described in an intuitive way.

As an exploratory technique of the data, we opted for documentary analysis, which can be a valuable technique for approaching qualitative data. This technique involves the analysis of documents that seeks to identify information from hypotheses raised.

According to Lüdke and André (2013, p. 45), "documents also constitute a powerful source from which evidence can be taken to support the researcher's assertions and statements".

Thus, we set out to select documents that would provide us with sufficient and reliable information to answer the guiding question of our research. And for this purpose, we searched for documents such as theses and dissertations in the databases of the Coordination for the Improvement of Higher Education Personnel (CAPES), particularly in the Catalog of Theses and Dissertations and in Capes Journal. The selection of the works considered the following search terms: "Mathematical Modeling", "Scenarios for Investigation", "Critical Mathematics Education" and the combination between them.

Among the criteria established for the selection of the works, we mention: works that contemplated the descriptors "Mathematical Modeling", "Scenarios for Investigation"; " Critical Mathematics Education", "Mathematical Modeling and Research Scenarios", "Mathematical Modeling and Critical Mathematics Education", works that contemplated mathematical disciplines in higher education; works that contemplated the construction of mathematical models; and research papers published in Portuguese and courses in Brazil.

The following is the number of searches found in both databases:

Table 3

Number of searches according to search terms

Search Terms	CAPES Thesis and Dissertation Catalog	CAPES Journal
Mathematical Modeling	7441	2133
Scenarios for Investigation	3784	2219
Critical Mathematics Education	651	1164
Mathematical Modeling and Scenarios for Research	25	9
Mathematical Modeling and Critical Mathematics Education	74	81

Source: elaborated by the author, 2024

Lopes and Pacheco (2023), identify a lack of empirical studies on Mathematical Modeling, which highlights the need for increased research in this field of investigation. In addition, the authors argue that, although there are literature review studies that analyze scientific production, few studies evidence the production of knowledge involving Mathematical Modeling from the perspective of Critical Mathematics Education in the context of higher education.

4.1 LITERATURE REVIEW

To identify the possible Scenarios for Investigation listed by Skovsmose (2000) and Alro and Skovsmose (2007), we searched the database of Catalogs of Theses and Dissertations and CAPES Journal, research related to different mathematical objects addressed in higher education and Mathematical Modeling. After selecting the researches, we chose to highlight the results pointed out in two works that show the role of Mathematical Modeling in the construction of different Scenarios for Investigation.

4.1.1 The First Scenario for Investigation

In a research carried out in the first semester of 2006 for students of a Geography course at a public university in Paraná, Araújo (2012) presents a Mathematical Modeling project guided by Critical Mathematics Education. In it, the author seeks to interpret the students' critical being from the proposal of the modeling activity. As a result of her analysis, the researcher found how the students interpreted what it means to be critical after their insertion in their realities. It points out that the evidence of different realities occurred after the support of these students in mathematical certainties to reach the conclusions of the project.

In the first stage of the project's development, Araújo (2012) presented the students with the discussion about what Mathematical Modeling is and how to approach it from a critical perspective. After the discussion, the students were asked to prepare a work plan for the project, define possible themes and form groups. Among the topics presented, the following are mentioned: Transposition of the São Francisco River, Physical impacts on the implementation of hydroelectric plants, socioeconomic aspects of the Green Line project in Belo Horizonte, Climate Legends, among others.

After the selection of the themes by the groups, the students were instructed to define the objective of the research and to understand how mathematics would be used in this project. The partial reports of the project of each group were presented each month, being a propitious moment for each one to receive guidance not only from the teacher/researcher but also from the other groups present. Araújo (2012) mentions that in all discussions he always sought to bring up his concern in relation to Critical Mathematics Education.

At the end of the semester, all projects were presented orally to the next class.

Araújo's (2012) research brings an interpretation of the critical approach of Mathematical Modeling in relation to the theme: socioeconomic aspects of the Green Line construction project in Belo Horizonte. Throughout the research, the author alternates her role between teacher and researcher depending on the events that occur in the classroom or outside it. However, he points out that at many times he had to play both roles concurrently.

Among the specific objectives of her research, she presents: to verify, through mathematical calculations, the veracity of the arguments and data presented by the government to justify the need for the Green Line project; ii) to estimate the useful life of the new viaduct built, in order to demonstrate how long the Green Line project will support the constant growth of the flow of vehicles in the delimited section.

In summary, it was necessary to identify the real need to build a viaduct and its impacts on the region under study.

To achieve the result of the research, the groups developed data collection through interviews with pedestrians and merchants in the region considered by the research so that they could report the inconveniences and expectations about the work from the mathematical study of traffic. At this point, the author highlights that the groups presented a critical attitude towards the theme and the reality of the local population at the end of the interviews. Among their perceptions, the students mention that there was no concern on the part of the government with the objective of creating projects that would mitigate the impacts of this work. After evidence of a critical awareness on the part of the students, there was a continuity in the mathematical treatment of the project.

With the help of a student from the university's Civil Engineering course, the group selected a series of formulas used in Traffic Engineering that estimated the useful life of the new viaduct built. At the end of the selection, the following expression was chosen: , where: C is the capacity of the stretch with continuous flow; W is the adjustment factor for lane width and side clearance, C is the adjustment factor for trucks, and Ô is the adjustment factor for buses.$C = 2000.w.c.\hat{o}$

Araújo (2012) exposes that the use of mathematical formulas or models and the questioning of their origin is desirable in a modeling approach according to Critical Mathematics Education. Among the desired questions is the origin of tables to establish the values of w, c and ô. It is worth noting that the group adopted the values provided by HCM-1965, without informing what this acronym meant. After the author's investigation, she discovers that it refers to the *Highway Capacity Manual,* [2] which corresponds to a manual and the main alternative for analyzing capacity and quality of service in Brazil for the evaluation of road infrastructure. Araújo (2012) also highlighted that the group did not bother to update the data in the table with values that were in accordance with their reality. The factors, and were adopted by the group, and were not questioned, leading to the result of vehicles/hour in each of the lanes of the viaduct. As the viaduct had three lanes, it was enough to calculate, arriving at the value of 5384 vehicles/hour as its maximum capacity.$w = 0{,}97c = 0{,}95\hat{o} = 0{,}97C = 2000 \times 0{,}97 \times 0{,}95 \times 0{,}97 = 1788C = 3 \times 1788$

After searching for information at the Belo Horizonte Transport and Transit Company (BHTRANS), the flow of vehicles at peak hours would be 3600 vehicles/hour, however, as it is old information offered by the company, the group chose to make an estimate in view of the lack of current information.

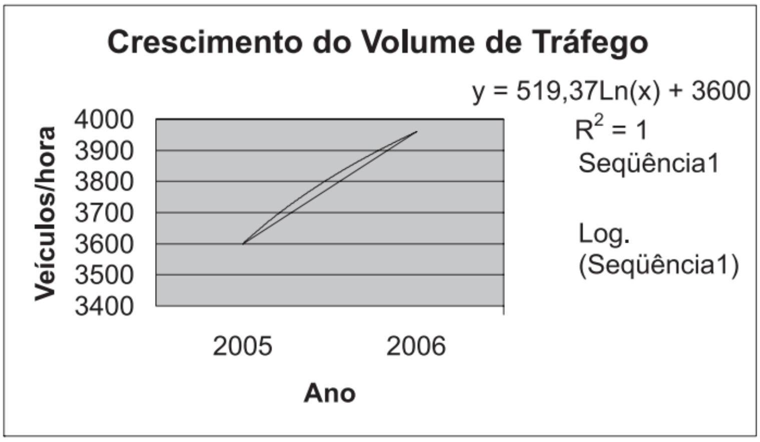
The questioning about the lack of information offered by public agencies was also perceived as a critical stance adopted by the group and was mentioned at various times in its reports.

In addition to the previous calculation, the group decided to add 10% to the flow of vehicles that will pass at the beginning of the implementation of the viaduct, however, he did not offer arguments to support this decision. There is an assumption on the part of Araújo (2012) that there were two values: that of 2005, with 3600 vehicles/hour; and the 2006 version, with vehicles/hour. According to the author, the choice of the two points occurred because the group intended to use the $3960 = (3600 + 360)$ *Excel software* to obtain a function that would allow an estimate of the useful life of the viaduct. With the use of the *software*, the group chose the trend line that would express the rapid growth of the flow of vehicles at the beginning of time with the subsequent decrease, since the traffic density would increase as a function of time. The trend line describes a linear regression of what is expected. Thus, the growth in traffic flow was expressed by the logarithmic function that allowed calculating the year in which the value of y approaches the capacity of the viaduct equal to 5364 vehicles/hour.$y = 519{,}37.\ln(x) + 3600$

[2] HCM is one of the most important references for transportation professionals. The document defines the capacity of an infrastructure as follows: "capacity of a facility is the maximum hourly rate, reasonably expected, at which people or vehicles cross a point or a uniform section of a lane or lane during a given period of time in a given condition of lane, traffic and operation"

Figure 1

Traffic Volume Growth between 2005 and 2006



Source: Araújo, 2012, p. 853

As a result, the maximum capacity of the viaduct would be reached in 29 years, since in 2005 the value of x in the function would be equal to 1. However, considering that the modeling project began in 2006, then the new calculation would take 27 years for the viaduct to reach the limit capacity.

Even though the arguments were built with imprecise estimates, the group's main motive was to reach conclusions from mathematical arguments.

According to Araújo (2012), the group's conclusion was quite frustrating, since they hoped to confirm the veracity of the calculations and data presented by the government that would justify the need for the Green Line Project. The author supports the hypothesis that, because they were students of a mathematical discipline and future geographers, the students felt supported by the traditional classes of mathematics teaching.

However, the group demonstrated a critical attitude by showing concern for the well-being of the population and the use of public money, something that mathematical calculations could not measure.

Araújo (2012) realizes that this theme is worth deepening from new research that investigates the role of Mathematical Modeling in Critical Mathematics Education.

4.1.2 The Second Scenario for Investigation

The second scenario we considered for our analysis adopted the methodology of Mathematical Modeling in the evaluation of the environmental impact caused by the discharge of a pollutant load into a river. The practice took place in the Calculus III discipline of an Environmental Engineering course at a public university in Paraná. The presented Mathematical Model describes the transport of a pollutant that can be solved numerically.

Oliveira and Pires (2020) point out that at the same time that Mathematical Modeling is a useful tool in assessing environmental impact and in decision-making, it is also a very motivating teaching methodology for students. In addition, they consider that the practice of Mathematical Modeling allows students to experience real situations, based on the analysis of real contexts and not just hypothetical situations.

As a process of interaction with the context, the authors make a brief presentation of the concept of transport of contaminants from rivers and some mathematical models involved. They add that in order to work with Mathematical Modeling, the teacher must be clear about the objectives to be achieved, such as motivation, recognition or the results of the models.

In order to clarify the practical usefulness of certain concepts presented in the discipline of Calculus III in the 1st semester of 2017, the authors adopted the pedagogical practice of Mathematical Modeling through the approach of problematizing themes related to the environment. The suggestion of the theme was presented by the same authors, and aimed to evaluate whether the pollution released by residents in the city of Pontal do Paraná can affect the quality of water and, consequently, tourism and the regional economy. It was noticeable that after the authors raised such questions, there was greater attention and participation of the students to the explanations and thus a change in attitude.

For Oliveira and Pires (2020), there are different mathematical models that describe physical, chemical, and biological processes that occur in water bodies such as rivers, lakes, and reservoirs and that represent the system of interest.

In order to study water quality, the authors considered the following parameters:

Dissolved Oxygen (DO): represents the condition of water quality;

Biochemical Oxygen Demand (BOD): establishes the degree of water pollution.

Thus, DO and BOD interact and their relationship is exposed by the equations that express the oxygen level of a river subjected to a pollutant:

$$\frac{dC_5}{dt} = -a_2.\left(\frac{C_6}{K_d+C_6}\right).C_5 - a_1.C_5 + R_5 \qquad (1)$$

$$\frac{dC_6}{dt} = -a_2.\left(\frac{C_6}{K_d+C_6}\right).C_5 - k_a.C_6 + R_6 \qquad (2)$$

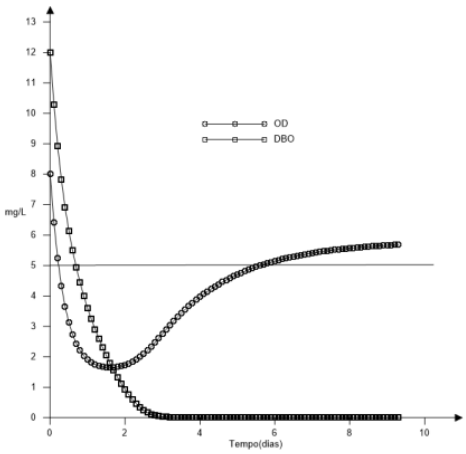
where, it represents the OD and represents the BOD.$C_5 C_6$[3]

According to the authors, there are eleven parameters that can be evaluated in the model, however, there was a restriction for the study of two of them, the one that represents the DO and the one that represents the BOD.$C_5 C_6$

The authors highlight that three classes were used, two theoretical and one in the laboratory for later analysis of the data through graphs generated by Excel. The validation stage of the model was proven based on scientific studies that proved its effectiveness (Cunha and Ferreira, 2006).

Figure 2

Comparison between DO and BOD



Source: Oliveira and Pires, 2020, p.11

As a result, the authors identified that the discharge of sewage into the river, given by BOD, causes a critical decay in the level of oxygen in the water. However, the oxygen level rises again after one day, stabilizing at around 6 mg/L. The students concluded that the water would only be adequate from the 5th day after contamination by the sewage discharge, after considering that the oxygen level must be above 5 mg/L for the water to be considered adequate. The degradation of the environment was also proven because, even after 10 days, the DO concentration that was 8 mg/L did not return to normal.

[3] For more information, see: https://www.scielo.br/j/csp/a/TDnPPt7Yzdbx8WDmXYK8nFg/?format=pdf&lang=pt

5 DISCUSSIONS

We present our analysis and discussions about the mathematical models employed in both scenarios discussed earlier.

We first consider the first Scenario for Investigation. In relation to the aspects and stages defined in the Mathematical Modeling, it is possible to affirm that the group, with the guidance of the teacher, followed the stages of Interaction, Mathematization and Mathematical Model following the ideas of Biembengut and Hein (2000). In this stage, there was a discussion of the role of Mathematical Modeling and its approach from a critical perspective. To this end, the group participated in the preparation of the work plan and defined the respective themes. In the Mathematization stage, the group defined the objective of the research and understood how mathematics would be used in this project. It was the moment when the hypotheses were raised and the variables *x* (year) and *y* (flow of vehicles/hour), the constants *w*, *c* and *ô* and the relationships described were investigated based on the expression investigated by the group members. The group's communication was a crucial factor for the investigation and decision-making. In the Mathematical Model stage, the implications of the solution found in the proposed model were analyzed. However, it did not obtain a significant degree of approximation and reliability with the problem-situation, since the values of the constants , and were not questioned. However, the previous calculation allows the discussion of other concepts related to local traffic capacity, such as how the variables are characterized and what are the parameters of the project, the study of the variation of annual, monthly, daily or hourly traffic, among other concepts. It will be up to the students, in case of interest, to deepen the theme.$C = 2000.w.c.\hat{o} w = 0{,}97 c = 0{,}95 \hat{o} = 0{,}97$

In addition, the group's simplified estimate, when considering the 10% increase in the flow of vehicles after one year of the implementation of the viaduct, did not bring arguments to support the calculations. As stated by Bassanezi (2012), when the model does not meet the needs, it is necessary to return to the Mathematization stage and verify the adjustment of hypotheses, variables, etc. However, it is worth highlighting the relevance of this problematization, in view of the study of the logarithmic function and linear regression.$y = 519{,}37.\ln(x) + 3600$

As pointed out by Araújo (2012), the objective of his research was to address Mathematical Modeling according to Critical Mathematics Education, a function that was fully successful in the discussions presented by the group in all stages of the project.

From the perspective of Skovsmose (2001), it is necessary to highlight the relevance of mathematical application and the elaboration of real mathematical models that relate important social activities. These models should provide *insights* into the hypotheses and an understanding of the entire process in society.

Thus, the proposal of the project was to abandon the exercise paradigm and propose a learning environment focused on Scenarios for Investigation that, according to Alro and Skovsmose (2010) are open and allow students to formulate questions, plan and participate in the investigation process. In the Green Line Project proposal, we identified a high degree of reference to the real world that corresponds to the learning environment of type (6) in the matrix established by the authors previously cited. We also highlight that the group sought to get closer to the respective purposes of the activity and assumed responsibility for the investigation process, from conducting interviews, formalizing the Mathematical Model studied and presenting the results to the other groups. In this way, we consider that there was effective communication based on cooperation and new forms of learning.

Regarding the second Scenario for Investigation, Oliveira and Pires (2020) show that the practice of Mathematical Modeling provided excellent results in relation to student performance, with a higher approval rate compared to previous semesters, ranging from 25 to 30% to 74%. Another point to be highlighted by them is that the students themselves requested complementary training that was offered as an extension course. At the end of the course, a group was formed in the area of environmental modeling.

In our analysis, we considered that all stages of Mathematical Modeling were considered from the perspective of Biembengut and Hein (2010), namely: (i) Interaction: there was the identification of the problem-situation, familiarization with the environmental context and the selection of mathematical models already validated by theoretical references; (ii) Mathematization: even in possession of the model validated by other researches, the students formulated the hypothesis and solved the problem in terms of the model. The parameters that represent the OD and the BOD were selected. This step requires knowledge about the mathematical entities used in the formulation of the model; (iii) Mathematical Model: the interpretation of the solution evidenced after analysis of the graph: BOD: there is a critical decay of the oxygen level in the river and the identification of the ideal level of oxygen in the water and the period in which the water would be contaminated. The validation of the model was presented by the theorists who founded the Mathematical Model presented in the

Interaction stage. In the Mathematical Model phase, the implications of the solution derived from the investigation were analyzed, as well as the evaluation of its relevance.$C_5 C_6$

In both scenarios presented, the Mathematical Modeling discussed as a teaching method reinforces the emphasis of mathematics for the student's education, greater development of problem-solving skills and greater interest in the applicability of mathematics.

6 CONCLUSION

In our article, we seek to highlight how Mathematical Modeling is related to different Scenarios for Investigation in a critical perspective of teaching and learning. Thus, we report research that deals with mathematical models and, in this way, allows us to leave the comfort zone employed by traditional practices in class and return to situations that encourage risk, uncertainties, dialogue and the search for solutions in a collaborative way. We are supporters of Skovsmose's (2000) ideas that scenarios for investigation are not the only solutions to be employed in a classroom, and that we should move between different learning environments, including the practice of exercises. In the same way, we point to the question posed by Skovsmose (2000, p. 58): *What are the possible gains of working in a risk zone associated with an investigation scenario?* However, there is still no clear distinction applied in class about the tradition of school mathematics and the tradition of investigative mathematics. We continue to search for answers to this question, as well as invite other researchers and enthusiasts on the subject to continue with this discussion.

Despite the bleak picture that shows that a good part of Mathematics Education still alternates environments (1) and (3), the challenge for researchers is to encourage not only the environment (6), but also to bring discussions about the learning environments of types (2) and (4). And one of the possibilities of discussion about the transition between the various environments can occur from Mathematical Modeling.

We realized that already consolidated mathematical models certify our veracity and, therefore, the validation stage of the model is restricted to its interpretation and analysis of the data. Thus, many existing models can serve as an important source of investigation and readjustment according to the research objectives. The setting and the choice of models will occur in the Interaction phase in which we make contact with the study context, as well as the theoretical knowledge that will support our analysis.

We emphasize that the study of both mathematical models allows for a deeper understanding of the themes, bringing to light more in-depth discussions about other related concepts, such as what is meant by traffic capacity, which variables and parameters should be considered, or even what are the interactions between the other substances in the proposed model to describe the transport of a pollutant. Therefore, the interdisciplinary role between concepts, disciplines and areas of knowledge becomes evident.

As presented in our research, an ideal model is one that not only explains the results but also predicts new results or unexpected situations or situations that do not match what was expected. We can consider that the group of the first scenario investigated had a series of doubts about the effectiveness of the values found, as well as failed to consider further explanations about some values found. However, these are doubts that instigate us to seek other and new answers so that we can further deepen the analysis of the aforementioned research as well as collaborate with the continuity of the study.

Although there is considerable interest in the results obtained in the mathematical models, in both scenarios, the researchers were interested in investigating how significant the critical posture of the students is in relation to the situations experienced, questioning the statements, the government's performance, the well-being of the local population and the use of public money.

Thus, we conclude that Mathematical Modeling has a significant role when it is desired to work with Investigation Scenarios of types (2), (4) and (6) as shown by the matrix of Alro and Skovsmose (2010) and Skovsmose (2000).

We leave it up to the reader to consider future research that portrays the transition between these learning environments, bringing their real contributions, perspectives and difficulties.

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