MEANINGFUL LEARNING IN CALCULUS TEACHING: THEORETICAL IMPLICATIONS FOR THE UNDERSTANDING OF THE CONCEPT OF DERIVATIVE

APRENDIZAGEM SIGNIFICATIVA NO ENSINO DE CÁLCULO: IMPLICAÇÕES TEÓRICAS PARA A COMPREENSÃO DO CONCEITO DE DERIVADA

APRENDIZAJE SIGNIFICATIVO EN LA ENSEÑANZA DEL CÁLCULO: IMPLICACIONES TEÓRICAS PARA LA COMPRENSIÓN DEL CONCEPTO DE DERIVADA

https://doi.org/10.56238/sevened2026.011-016

Juliana de Fatima Holm Brim[1], Bianca Aparecida Holm de Oliveira[2], Nilcéia Aparecida Maciel Pinheiro[3]

ABSTRACT

Considering the high failure rates in Differential and Integral Calculus courses in higher education, often associated with pedagogical practices centered on procedural memorization and repetitive exercise solving, it becomes necessary to discuss theoretical approaches that promote conceptual understanding. This article aims to discuss the foundations of the Theory of Meaningful Learning and to analyze its implications for the teaching of Differential and Integral Calculus, with emphasis on derivatives, mechanical learning, and assimilation processes. To this end, a theoretical study is conducted, characterized as a theoretical excerpt from a doctoral research developed by one of the authors, grounded mainly in the assumptions of Ausubel, Novak, and Hanesian, as well as contributions by Moreira. Thus, it is observed that meaningful learning, by prioritizing prior knowledge, hierarchical organization of concepts, and intentional pedagogical mediation, enables the overcoming of mechanistic practices and fosters conceptual understanding in Calculus teaching. The results indicate that pedagogical strategies aligned with this theory favor retention, transfer, and application of mathematical concepts in different contexts. It is concluded that the Theory of Meaningful Learning constitutes a consistent framework for re-signifying the teaching of Differential and Integral Calculus, contributing to more lasting, critical, and integrated learning in higher education.

Keywords: Meaningful Learning. Calculus Teaching. Derivatives. Mechanical Learning. Higher Education.

RESUMO

Considerando os elevados índices de reprovação no ensino de Cálculo Diferencial e Integral no ensino superior, frequentemente associados a práticas pedagógicas centradas na

[1] Dr. in Science and Technology Teaching. Universidade Federal do Paraná (UTFPR).
E-mail: julianafhbrim@gmail.com Orcid: https://orcid.org/0000-0002-1258-9276
[2] Doctoral student in Science and Technology Teaching. Universidade Tecnológica Federal do Paraná (UTFPR). E-mail: biaholm2018@gmail.com Orcid: https://orcid.org/0000-0002-8814-4150
[3] Dr. in Science and Technology Education. Universidade Tecnológica Federal do Paraná (UTFPR).
E-mail: nilceia@utfpr.edu.br Orcid: https://orcid.org/0000-0003-3313-1472

memorização de procedimentos e na resolução mecânica de exercícios, torna-se necessário discutir abordagens teóricas que favoreçam a construção de significados conceituais. Objetiva-se, neste artigo, discutir os fundamentos da Teoria da Aprendizagem Significativa e analisar suas implicações para o ensino de Cálculo Diferencial e Integral, com ênfase nos conceitos de derivadas, aprendizagem mecânica e processos de assimilação. Para tanto, procede-se a um estudo de natureza teórica, caracterizado como um recorte de uma pesquisa de doutorado desenvolvida por uma das autoras, fundamentado principalmente nos pressupostos de Ausubel, Novak e Hanesian, bem como em contribuições de Moreira. Desse modo, observa-se que a aprendizagem significativa, ao priorizar os conhecimentos prévios, a organização hierárquica dos conceitos e a mediação pedagógica intencional, possibilita superar práticas mecanicistas e promover a compreensão conceitual no ensino de Cálculo. Os resultados evidenciam que estratégias pedagógicas alinhadas a essa teoria favorecem a retenção, a transferência e a aplicação dos conceitos matemáticos em diferentes contextos. Conclui-se que a Teoria da Aprendizagem Significativa constitui um referencial consistente para ressignificar o ensino de Cálculo Diferencial e Integral, contribuindo para aprendizagens mais duradouras, críticas e integradas no ensino superior.

Palavras-chave: Aprendizagem Significativa. Ensino de Cálculo. Derivadas. Aprendizagem Mecânica. Ensino Superior.

RESUMEN

Considerando los elevados índices de reprobación en las asignaturas de Cálculo Diferencial e Integral en la educación superior, frecuentemente asociados a prácticas pedagógicas centradas en la memorización de procedimientos y en la resolución repetitiva de ejercicios, se hace necesario discutir enfoques teóricos que favorezcan la comprensión conceptual. Este artículo tiene como objetivo discutir los fundamentos de la Teoría del Aprendizaje Significativo y analizar sus implicaciones para la enseñanza del Cálculo Diferencial e Integral, con énfasis en las derivadas, el aprendizaje mecánico y los procesos de asimilación. Para ello, se desarrolla un estudio teórico, caracterizado como un recorte teórico de una investigación doctoral realizada por una de las autoras, fundamentado principalmente en los aportes de Ausubel, Novak y Hanesian, así como en las contribuciones de Moreira. De este modo, se observa que el Aprendizaje Significativo, al priorizar los conocimientos previos, la organización jerárquica de los conceptos y la mediación pedagógica intencional, permite superar prácticas mecanicistas y favorece la comprensión conceptual en la enseñanza del Cálculo. Los resultados indican que las estrategias pedagógicas alineadas con esta teoría contribuyen a la retención, transferencia y aplicación de los conceptos matemáticos en distintos contextos. Se concluye que la Teoría del Aprendizaje Significativo constituye un marco teórico consistente para resignificar la enseñanza del Cálculo Diferencial e Integral en la educación superior.

Palabras clave: Aprendizaje Significativo. Enseñanza del Cálculo. Derivadas. Aprendizaje Mecánico. Educación Superior.

1 INTRODUCTION

The learning of concepts involved in Differential and Integral Calculus, in higher education, has been marked by difficulties that result in high failure rates in this discipline. Such challenges are often associated with pedagogical practices centered on the memorization of procedures and techniques of derivation and integration, based on lists of repetitive exercises, little conceptually articulated and detached from previous knowledge and the students' reality. Intentional pedagogical mediation, combined with the conscious use of educational resources and technologies, has been pointed out as a fundamental element to promote more meaningful and contextualized learning in higher education, contributing to overcoming practices focused exclusively on the repetition of procedures (Veiga et al., 2025)

In this context, it is pertinent to discuss theoretical approaches that make it possible to understand and overcome this scenario, favoring meaningful learning and evidencing the versatility and potentiality of Differential and Integral Calculus.Among the theories that offer consistent subsidies for this reflection, the Theory of Meaningful Learning, proposed by David Ausubel, stands out, which emphasizes the importance of the learner's prior knowledge as an anchorage for new knowledge. By considering that the single most important factor for learning is what the student already knows, this perspective shifts the focus of teaching from the simple transmission of content to conceptual organization and intentional pedagogical mediation, recognizing that the assimilation of knowledge occurs in a structured and hierarchical way.

This article presents a theoretical excerpt from a doctoral research developed by one of the authors, in which the contributions of pedagogical strategies based on Meaningful Learning for the teaching of Derivatives in a Software Engineering course are investigated. The excerpt presented here aims to discuss the theoretical foundations of meaningful learning, contrasting it with mechanical learning, as well as to analyze its implications for the teaching of Differential and Integral Calculus.

Thus, the text is organized into four sections: initially, the theoretical foundation of Meaningful Learning is presented; then, the distinction between mechanical and meaningful learning in the context of the teaching of Calculus is discussed; then, the types of meaningful learning and the assimilation process are analyzed; finally, the pedagogical implications of this approach for the teaching of Differential and Integral Calculus are discussed, culminating in the final considerations.

2 MEANINGFUL LEARNING

Understanding the challenges related to the teaching of Differential and Integral

Calculus requires, initially, the explanation of the theoretical framework that supports the analyses developed in this study. In this sense, the Theory of Meaningful Learning offers consistent foundations to discuss how mathematical knowledge is organized, assimilated and mobilized by students. By prioritizing the cognitive processes involved in the construction of meaning, this theory enables a critical reading of traditional pedagogical practices and guides the proposition of didactic strategies that are more coherent with the conceptual nature of Calculus.

2.1 THEORY OF MEANINGFUL LEARNING

The Theory of Meaningful Learning, proposed by David Ausubel (1968, 1973), is part of the field of cognitivist theories and seeks to explain the way individuals acquire, organize and retain knowledge. For the author, learning occurs in a significant way when new information is related, in a substantive and non-arbitrary way, to preexisting concepts in the learner's cognitive structure, called subsumers.

Ausubelian conceptions distance themselves from mechanistic perspectives by understanding learning as a complex cognitive process, which is not limited to repetition or arbitrary memorization of contents. From this perspective, learning implies the substantive and non-literal interaction between new knowledge and that already present in the learner's cognitive structure. According to Ausubel, Novak and Hanesian (1998), this structure is organized in a hierarchical way, consisting of more general and inclusive concepts, which function as subsumers and enable the anchoring, assimilation and reorganization of more specific ideas.

In the teaching of Mathematics, and particularly in Differential and Integral Calculus, this perspective implies recognizing that the understanding of concepts such as limit, derivative and rate of variation depends on the existence of well-structured prior knowledge, such as functions, graphs and notions of variation. Thus, the role of the teacher shifts to the identification of this previous knowledge and to the organization of teaching in order to favor progressive and coherent conceptual relations. (Brim, 2024)

2.2 MACHINE LEARNING AND MEANINGFUL LEARNING IN CALCULUS TEACHING

The theory of meaningful learning has as its central focus cognitive learning, understood as the result of the organization and reorganization of information in the learner's cognitive structure. This structure is formed by a hierarchical set of concepts, ideas and propositions, which are modified as new information is incorporated in a non-arbitrary and non-literal way (Ausubel, 2003). In the context of teaching Differential and Integral Calculus, this perspective becomes particularly relevant, since the concepts covered in this discipline require high levels of abstraction and depend heavily on previously consolidated mathematical knowledge.

As opposed to meaningful learning, Ausubel defines machine or automatic learning as that in which new information is stored in a literal, arbitrary manner and disconnected from relevant concepts already existing in the student's cognitive structure (Ausubel, 2003; Moreira, 2019). In the teaching of Calculus, typical examples of this type of learning can be observed when the student memorizes derivation rules, algebraic techniques or operational procedures without understanding the concepts that underlie them. The memorization of derivative formulas, the automatic application of the product or chain rule, as well as the mechanical resolution of integrals by substitution, without understanding the geometric, analytical meaning or real applications, configure recurrent situations of machine learning.

In addition, study practices based on "last-minute" learning, with an exclusive focus on performance in assessments, also configure rote learning. In these situations, the student can even reproduce procedures correctly in a short period of time, however, as pointed out by Ausubel (2003), it tends to be quickly forgotten, not contributing to the construction of a solid conceptual basis necessary for progression in more advanced contents of Differential and Integral Calculus.

However, rote learning should not be understood as completely dissociated from meaningful learning, but rather as one of the extremes of a *continuum* (PRAIA, 2000). In the initial process of appropriating new knowledge, especially in areas completely new to the learner, rote learning can play a provisional role. In the teaching of Calculus, for example, when coming into contact for the first time with limit notation, with the symbology of differential calculus or with certain formal definitions, it is natural for the student to initially carry out a more literal learning, until relevant concepts come into existence in their cognitive structure and can function as subsumers. (Novak, 1977; Moreira, 2019).

Meaningful learning, in turn, occurs when the concepts of Differential and Integral Calculus come to be understood in relation to existing previous knowledge, in a substantive and non-arbitrary way. A classic example is the learning of the concept of derivative, which acquires meaning when the student is able to relate it to previously constructed ideas, such as the concept of function, graphical interpretation, the average rate of change and the slope of a line. In this case, the derivative is no longer just an algebraic expression or an operational rule and is understood as the limit of the instantaneous rate of change, associated with the local behavior of the function. (Ausubel, 2003; Moreira, 2019).

Similarly, in the study of integrals, meaningful learning occurs when the student understands the integral defined not only as the result of the application of a calculation technique, but as a measure associated with the area under the graph of a function, related to problems of accumulation, variation, and continuous sum. When these relationships are established, new knowledge begins to be anchored in relevant subsumers, promoting the assimilation and reorganization of the cognitive structure, which favors both the retention and the possibility of applying the concepts in different mathematical and interdisciplinary contexts.

In this sense, the role of the teacher in the teaching of Differential and Integral Calculus becomes central, since it is up to him to identify the students' previous knowledge, map the availability of relevant subsumers and organize teaching in order to favor the transition from rote learning to meaningful learning. As Moreira (2019) points out, learning only becomes effectively meaningful when the material is potentially meaningful and when the learner manifests a willingness to establish substantive relationships with what he or she already knows. Thus, the teaching of Calculus, based on the Ausubelian theory, requires pedagogical intentionality, conceptual planning and strategies that privilege understanding, and not only the reproduction of procedures, which raises an alert regarding the use of exercise lists as the only didactic resource, a practice that is still recurrent among teachers of this discipline.

2.3 TYPES OF MEANINGFUL LEARNING AND THE PROCESS OF ASSIMILATION

Within the scope of the Theory of Meaningful Learning, Ausubel, Novak and Hanesian (1980, 1983) establish important distinctions between the types of meaningful learning and the cognitive processes involved in the acquisition of knowledge, which allows us to understand, in greater depth, how mathematical concepts are structured and reorganized in the mind of the learner. These distinctions are particularly relevant in the teaching of Differential and Integral Calculus, an area in which highly abstract concepts require well-established cognitive relationships.

Ausubel (2003) defines three types of meaningful learning: representational, conceptual, and propositional. Representational learning consists of attributing meanings to symbols, being the most elementary form of meaningful learning. Although it comes close to memorization, it is not arbitrary, as the symbol comes to represent an object or idea with psychological meaning for the learner. In the context of Calculus, one can consider, for example, the initial familiarization with symbols such as , , or . These symbols, in isolation, do not constitute concepts, but acquire meaning as the student understands what they represent in the mathematical field, establishing a basis for later learning.$f(x) lim f'(x)$

Concept learning, on the other hand, represents a more elaborate level and involves understanding regularities, properties, and characteristics common to a class of objects or phenomena. In the teaching of Differential and Integral Calculus, this learning is manifested, for example, in the construction of the concept of function, which starts to act as a subsumer for the understanding of ideas such as limit, continuity, rate of variation and derivative. According to Ausubel (2003), the formation of concepts occurs through processes of progressive differentiation and generalization, in which the student formulates hypotheses, tests properties and cognitively reorganizes the meanings attributed to mathematical objects.

Propositional learning involves the understanding of ideas expressed through propositions, going beyond the simple sum of the meanings of the words or symbols involved. In this type of learning, the focus is on the relationship between concepts. A classic example in Calculus is the formal definition of derivative as the limit of the average rate of change. In order for this proposition to be learned meaningfully, the student needs to understand the concepts of function, variation, limit, and incremental ratio, establishing a non-arbitrary and non-literal relationship between them. According to Moreira (2019), propositional learning is central to higher education, as it is at this level that laws, theorems, and mathematical definitions are structured.

Regardless of the type, Ausubel (2003) points out that meaningful learning corresponds only to the first phase of a broader cognitive process, called the assimilation process. This process explains how potentially significant new ideas interact with relevant concepts already existing in the cognitive structure, called subsumers. Assimilation occurs when new knowledge is anchored in a subsumption, modifying it and making it more inclusive and differentiated.

In the teaching of Differential and Integral Calculus, this process can be clearly observed when the concept of limit, previously established, acts as a subsumer for learning the definition of derivative. Initially, the new information (definition of derivative) is anchored to the concept of limit, producing an emergent meaning that results from the interaction between both. This interactional product, according to Ausubel (2003), represents a fundamental stage of significant learning, as both the new concept and the subsumer undergo cognitive modifications.

With the advancement of learning, what Ausubel calls obliterating assimilation occurs, a natural process in which the specific details of the new knowledge become progressively less dissociable from the subsumer that gave it meaning. In the example of derivatives, as the student starts to use operative rules for the calculation of derivatives, the explicit use of the definition via limit tends to be abandoned. However, this does not characterize a learning loss, but a cognitive reorganization that favors the retention of more general and functional ideas. According to Ausubel (2003), forgetting, in this context, is not total, but residual, allowing for rapid relearning when necessary.

In addition to the process of assimilation, Ausubel distinguishes three forms of meaningful learning: subordinate, superordinate, and combinatorial. Subordinate meaningful learning is the most frequent and occurs when new information is anchored in more general concepts that already exist. In calculus, the learning of derivatives of specific functions, such as exponentials or logarithmics, can be understood as derivative subordinate, as long as the general concept of derivative is well established. Correlative subordinate learning, on the other hand, occurs when the new concept expands or modifies the subsumption, as in the case of partial derivatives, which redefine and expand the previously constructed concept of derivative.

Superordinate meaningful learning manifests itself when a broader concept is constructed from more specific ideas already existing in the cognitive structure. In the context of Calculus, this can occur when the student understands that concepts such as rate of change, slope of the tangent line, growth and decrease of functions are subordinated to a more general concept of derivative. According to Moreira (2006), the cognitive structure is dynamic, allowing alternations between subordinate and superordinate learning throughout the educational process.

Finally, combinatorial meaningful learning occurs when new knowledge does not subordinate itself or pre-existing concepts, but relates to them at the same hierarchical level. Although more complex and difficult to learn initially, this form of learning is common in advanced Calculus content, such as the geometric and analytical interpretation of different integration techniques, which require the simultaneous articulation of multiple previously constructed concepts.

Thus, the types and forms of meaningful learning, articulated with the assimilation process, allow us to understand that the teaching of Differential and Integral Calculus demands more than the presentation of algorithmic procedures. It requires, above all, the conceptual organization of the contents, the identification of relevant subsumers and the creation of conditions for the student to establish substantive cognitive relationships, favoring the construction of lasting meanings.

3 IMPLICATIONS FOR THE TEACHING OF DIFFERENTIAL AND INTEGRAL CALCULUS

Considering the assumptions of the Theory of Meaningful Learning, the teaching of Differential and Integral Calculus cannot be understood as a process centered exclusively on the transmission of formulas and techniques. Knowing that in the Ausubelian perspective, the learning of new knowledge occurs in a significant way when it relates, in a non-arbitrary and non-literal way, with relevant concepts already existing in the learner's cognitive structure, in the context of Calculus, this implies recognizing that concepts such as functions, limits and variation are indispensable subsumers for the understanding of more complex ideas, as derivatives and integrals. (Brim, 2024)

From this perspective, it becomes evident that pedagogical practices based predominantly on extensive lists of exercises, focused on the mechanical repetition of procedures, tend to favor machine learning. Although such practices can play a specific role in technical consolidation, they are insufficient when assumed as a central teaching strategy. As discussed by Ausubel (2003) and Moreira (2019), the reproduction of algorithms, dissociated from conceptual understanding, does not guarantee the construction of meanings and compromises the retention and transfer of knowledge to new situations.

In the teaching of Differential and Integral Calculus, the pedagogical intentionality is manifested, above all, in the conceptual organization of the contents and in the way they are presented to the students. Ausubel (2003) highlights progressive differentiation as an instructional principle, arguing that the most general and inclusive concepts should be introduced initially, being progressively differentiated in terms of specificity. Applied to calculus, this principle suggests that the approach to derivatives should start from broad ideas, such as rate of change and global behavior of functions, before moving on to formal definitions and specific calculus techniques.

In this sense, the previous organizers play a fundamental role in the process of teaching and learning Calculus. According to Ausubel, Novak and Hanesian (1998), these introductory materials work as cognitive bridges between the student's previous knowledge and the new content to be learned. Discussions on function graphs, growth and degrowth analysis, as well as the use of technological resources, such as graphic visualization software, can act as preliminary organizers for the study of derivatives, favoring the construction of meanings before mathematical formalization.

In addition, the implications of meaningful learning for the teaching of Calculus highlight the centrality of the teacher's role. According to Moreira (2019), it is up to the teacher to identify the subsumers available in the students' cognitive structure, diagnose conceptual gaps and plan strategies that favor the interaction between previous knowledge and new content. In the case of derivatives and integrals, this implies recognizing recurrent difficulties, such as the understanding of limits or the geometric interpretation of concepts, and proposing didactic situations that promote the integrative reconciliation of these meanings.

Another relevant aspect refers to the understanding of the process of obliterating assimilation in the teaching of Calculus. According to Ausubel (2003), forgetting should not be understood as a failure of the educational process, but as a natural phenomenon of meaningful learning. In the context of derivatives, for example, it is expected that the student will no longer resort to the formal definition via limit to use consolidated operative rules. This process indicates that the concept has been assimilated and integrated into the cognitive structure, becoming functional for subsequent learning.

Thus, the implications of the Theory of Meaningful Learning for the teaching of Differential and Integral Calculus point to the need for pedagogical practices that value conceptual understanding, hierarchical organization of contents, and intentional teacher mediation. Teaching Calculus, in this perspective, means promoting conditions for the student to understand the meaning of mathematical concepts, establish relationships between them and use them critically and consciously, overcoming a fragmented and mechanistic view of mathematical knowledge.

4 FINAL CONSIDERATIONS

The discussion developed in this article shows that the Theory of Meaningful Learning constitutes a consistent theoretical framework to rethink the teaching of Differential and Integral Calculus, especially with regard to overcoming practices collected from mechanical learning and the promotion of conceptual, lasting and meaningful learning. By emphasizing the interaction between new knowledge and the student's previous cognitive structure, this approach highlights the role of intentional pedagogical mediation in the organization of teaching.

The distinction between mechanical and meaningful learning, as well as the understanding of the types of meaningful learning and the assimilation process, allow the teacher to critically analyze their practices and plan interventions that are more coherent with the formative objectives of higher education. In the context of Calculus, this implies valuing strategies that favor the conceptual understanding, visualization and contextualization of the contents and recognition of their potentiality and versatility.

Finally, by presenting a theoretical excerpt from a doctoral research, this work reinforces the relevance of Meaningful Learning as a foundation for investigations and pedagogical practices in the teaching of Mathematics. It is hoped that the reflections proposed here will contribute to the development of more effective didactic approaches and to the construction of truly meaningful mathematical learning.

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